

## Timing and Amplitude Tolerances for Crab Cavity

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### Methodology:

The purpose of this analysis is to define LLRF timing and amplitude jitter tolerances for the crab cavity and to define beam timing and energy jitter tolerances in the presence of a crab cavity. The only criterion used in this report is loss of luminosity – a luminosity loss of 2% is considered the acceptable limit for determining the tolerance of a single parameter. The BDS lattice used in the calculation for 20 mrad crossing angle was obtained from ref. [1]. The BDS lattice used in the calculation for 14 mrad crossing angle was obtained from ref. [2]. The calculational tool used for tracking is MAD8.23. An idealized crab cavity is simulated with a 1st order matrix element  $R_{25}$  ( $\delta x' = R_{25} * \delta(ct)$ ) inserted into the lattice file at the location of the crab cavity. For the 20 mrad case  $R_{25} = -0.000614256$ , and for the 14 mrad case  $R_{25} = -0.00042683$ . This makes no assumption about the type of cavity, or at what frequency it is operated. The program Guinea Pig [3] is used for calculating luminosity.

For the beam timing and energy jitter calculations, each data point is the average over 30 seeds. For each seed a distribution of 20000 particles (representing 10000 collisions) is tracked from the front of the BDS to the IP. Each seed represents a sample from the gaussian distributed beam timing or energy error distribution. The  $e^+$  and  $e^-$  beam jitters are uncorrelated.

For the crab cavity timing jitter calculations, each data point is the average over 40 seeds. For each seed the same distribution of 20000 particles is used (with no beam timing or energy error). Each seed represents a sample from the gaussian distributed cavity timing error distribution. The horizontal position error at the IP is given by  $\delta x = R_{25} \times R_{12} \times c \times \delta t$ , where  $R_{25}$  is given above,  $R_{12} = 16.36$  is the 1st order matrix element ( $\delta x_{IP} = R_{12} * \delta x'_{CC}$ ) between the crab cavity and the IP, and  $c$  is the speed of light.

For the crab cavity amplitude jitter calculations, each data point is the average over 30 seeds. For each seed the same distribution of 20000 particles is used (no beam timing or energy error). Each seed represents a sample from the gaussian distributed cavity amplitude error distribution. The horizontal position error at the IP is given by  $\delta x = R_{12} \times R_{25} \times \Delta(ct) \times \delta A / A$ , where  $\Delta(ct)$  is the particle longitudinal position within the bunch, and  $\delta A / A$  is the relative cavity amplitude error.

Because of the strong beam-beam effect at the IP, luminosity is not maximized when the collision point is located exactly at the waists of the final foci [4]. Luminosity is maximized when the final foci are 240 $\mu$ m upstream of the collision point, and this number is used in the Guinea Pig simulations.

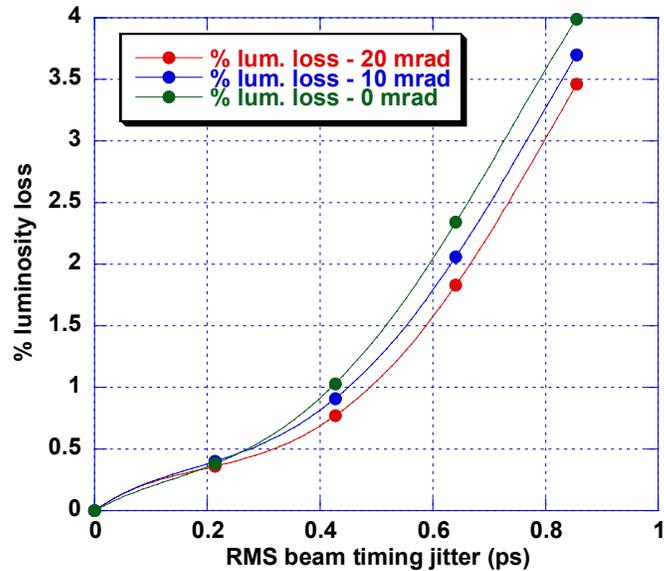
The nominal ILC beam parameters are used in the calculations. The initial distribution is thrown gaussian in all dimensions with  $\sigma_{ct} = .3$ mm and  $\sigma_{p/p} = 7E-4$  (6E-4 for the 14 mrad case). Transverse gaussian distributions are also thrown, with  $\gamma_{\epsilon_x} = 10E-6$  and  $\gamma_{\epsilon_y} = 4E-8$ . All distributions are cut off at  $\pm 3.9\sigma$ , including the error distributions. Twelve different error configurations are examined.

- 1-3) Beam timing errors for 0, 10, and 20 mrad xing angles.
- 4) Beam energy errors for 20 mrad xing angle.

- 5-6) Cavity timing errors for 20 mrad xing angle;  $e^+/e^-$  cavity errors uncorrelated and anticorrelated.
- 7) Cavity amplitude errors for 20 mrad xing angle;  $e^+/e^-$  cavity errors uncorrelated.
- 8) Beam timing errors for 14 mrad xing angle.
- 9) Beam energy errors for 14 mrad xing angle.
- 10-11) Cavity timing errors for 14 mrad xing angle;  $e^+/e^-$  cavity errors uncorrelated and anticorrelated.
- 12) Cavity amplitude errors for 14 mrad xing angle;  $e^+/e^-$  cavity errors uncorrelated.

Results:

Figures 1-4 show the results for beam timing and energy jitter, and for cavity timing and amplitude jitter for the 20 mrad case. Figures 5-8 show the results for beam timing and energy jitter, and for cavity timing and amplitude jitter for the 14 mrad case. Figure 5 is a graphical illustration of why beam timing jitter and cavity timing jitter have very different tolerances. Table 1 is a summary of the tolerances for the 20 mrad case, and Table 2 is a summary of the tolerances for the 14 mrad case.



**Figure 1: % luminosity loss vs beam timing jitter for 0, 10, and 20 mrad crossing angles. The lines are quartic fits.**

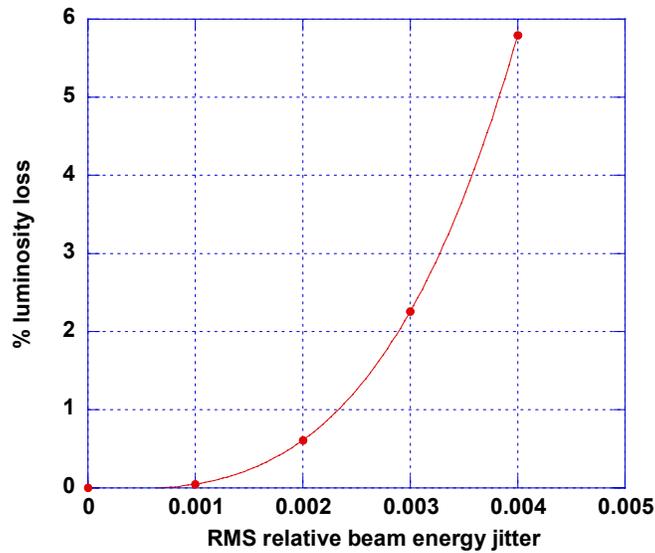


Figure 2: % luminosity loss vs beam energy jitter for 20 mrad crossing angle. The line is a quartic fit.

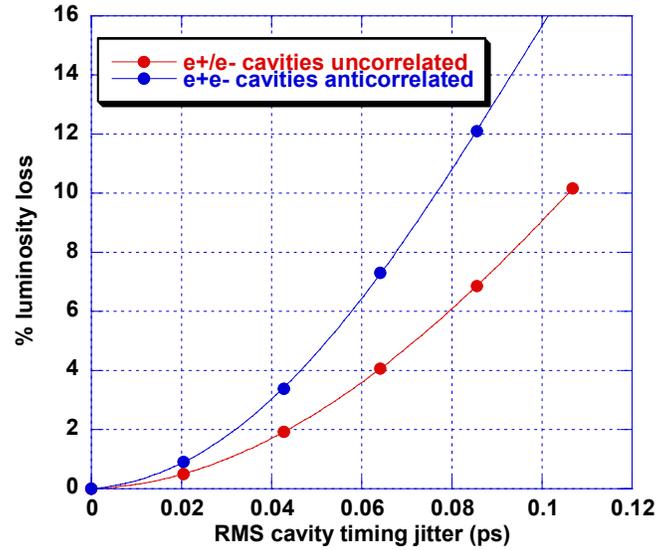


Figure 3: % luminosity loss vs crab cavity timing jitter for 20 mrad crossing angle. The lines are quartic fits.

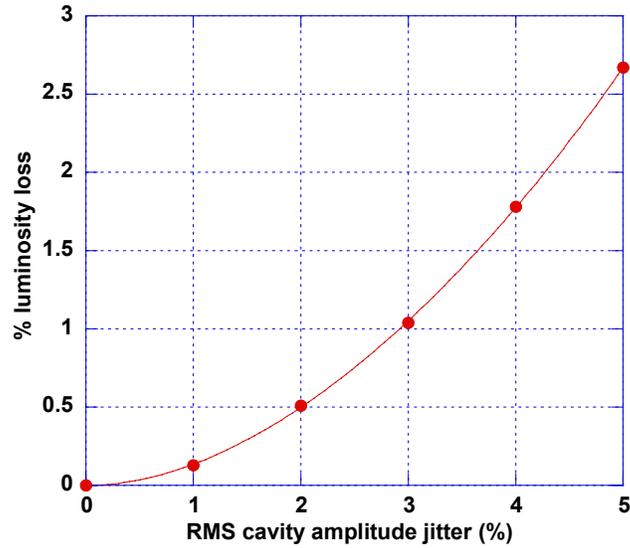


Figure 4: % luminosity loss vs crab cavity amplitude jitter for 20 mrad crossing angle. The line is a quartic fit.

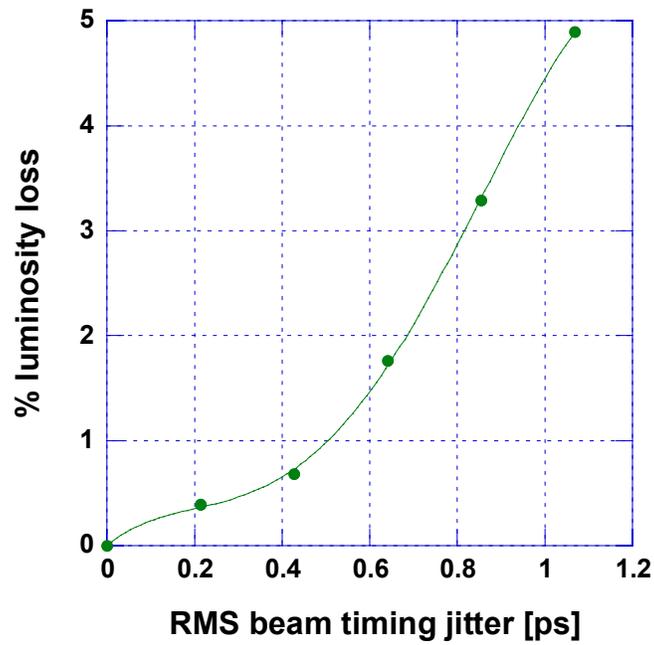


Figure 5: % luminosity loss vs beam timing jitter for 14 mrad crossing angle. The line is a quartic fit.

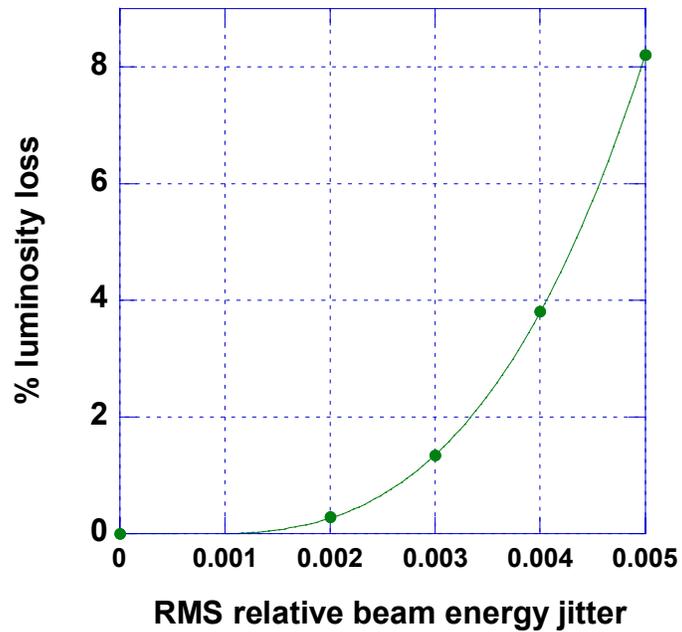


Figure 6: % luminosity loss vs beam energy jitter for 14 mrad crossing angle. The line is a quartic fit.

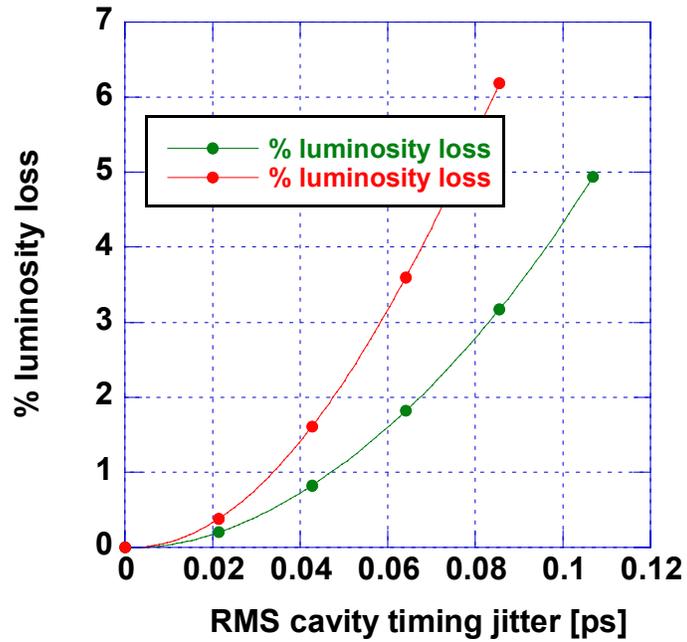


Figure 7: % luminosity loss vs crab cavity timing jitter for 14 mrad crossing angle. The lines are quartic fits.

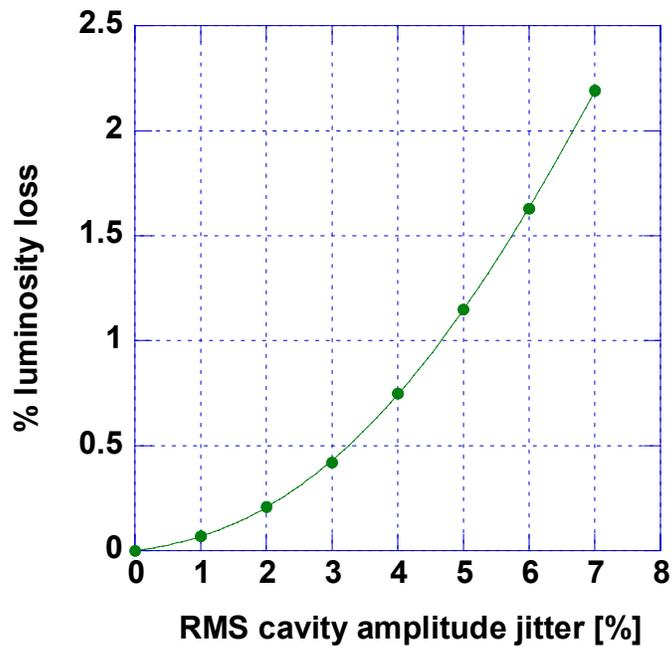


Figure 8: % luminosity loss vs crab cavity amplitude jitter for 14 mrad crossing angle. The line is a quartic fit.

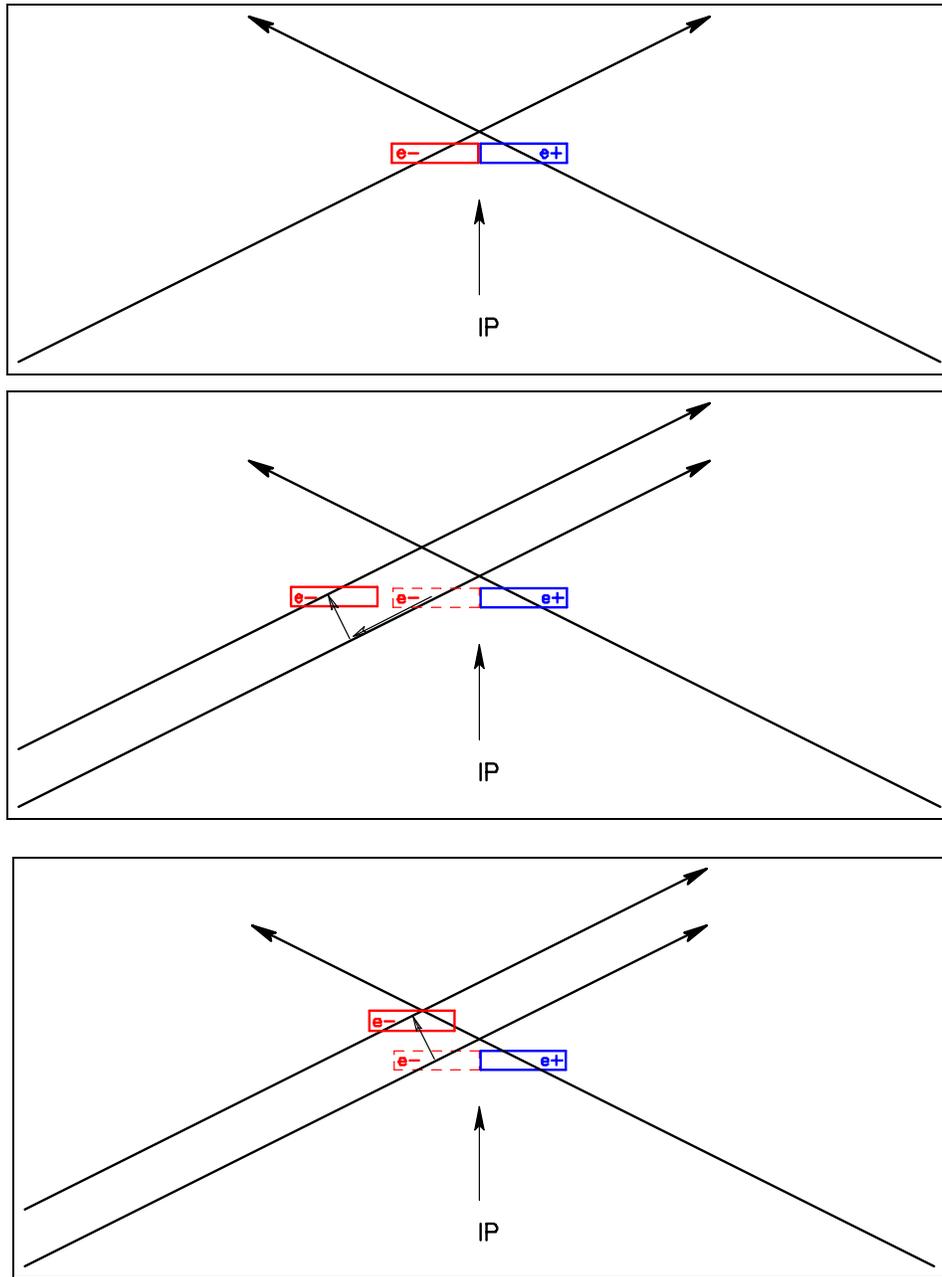


Figure 9: The top illustration shows a perfectly timed collision between an  $e^+$  and  $e^-$  bunch. The middle plot shows a collision between a perfectly timed  $e^+$  bunch and a late  $e^-$  bunch. The overlap integral is the same, but the collision is offset in  $z$ . The bottom plot shows a collision between a perfectly timed  $e^+$  bunch and a perfectly timed  $e^-$  bunch but with crab cavity timing error. The overlap integral is reduced, and the collision is offset in  $z$ . The vertical axis is  $x$ , and the horizontal axis is  $z$ , and they are not to scale.

	<b>timing tolerance limiting luminosity loss to &lt; 2% (ps)</b>	<b>amplitude tolerance limiting luminosity loss to &lt;2% (%)</b>
<b>RMS beam timing jitter</b>	<b>0.67</b>	
<b>RMS beam energy jitter</b>		<b>0.29</b>
<b>RMS cavity timing jitter (uncorrelated)</b>	<b>0.043</b>	
<b>RMS cavity timing jitter (anticorrelated)</b>	<b>0.032</b>	
<b>RMS cavity amplitude jitter</b>		<b>4.3</b>

**Table 1: Summary of tolerances for crab cavity timing and amplitude control, and beam timing and energy for 20 mrad crossing angle. Considered separately, each of these tolerances will limit the average luminosity loss to <2%. These tolerances are taken from Figures 1-4. They are the intersection of the quartic fits with 2% on the vertical axis.**

	<b>timing tolerance limiting luminosity loss to &lt; 2% (ps)</b>	<b>amplitude tolerance limiting luminosity loss to &lt;2% (%)</b>
<b>RMS beam timing jitter</b>	<b>0.68</b>	
<b>RMS beam energy jitter</b>		<b>0.33</b>
<b>RMS cavity timing jitter (uncorrelated)</b>	<b>0.067</b>	
<b>RMS cavity timing jitter (anticorrelated)</b>	<b>0.047</b>	
<b>RMS cavity amplitude jitter</b>		<b>6.6</b>

**Table 2: Summary of tolerances for crab cavity timing and amplitude control, and beam timing and energy for 14 mrad crossing angle. Considered separately, each of these tolerances will limit the average luminosity loss to <2%. These tolerances are taken from Figures 5-8. They are the intersection of the quartic fits with 2% on the vertical axis.**

References:

- [1] <http://www.slac.stanford.edu/~mdw/ILC/2006b/>
- [2] <http://www.slac.stanford.edu/~mdw/ILC/2006e/>
- [3] author D. Schulte; downloaded from <http://hepwww.ph.qmul.ac.uk/~white/accodes/>
- [4] M. Church, ILC DocDB-263, <http://docdb.fnal.gov>