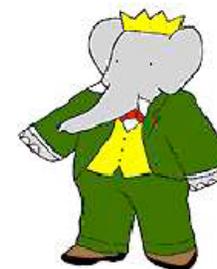


Charm Review

Jim Wiss
Univ of Illinois
APS meeting
May 2 , 2004

- Update on charm mixing
- Charm semileptonic decay
 - Analysis of $D \rightarrow K^* \mu \nu$
 - Analysis of $D_s \rightarrow \phi \mu \nu$
- Charm 3 body hadronic decay
 - Isobar model versus K- matrix
 - Dalitz analysis as probes of new physics
- Excited charm spectroscopy
 - cu and cd results from Belle and Focus
 - cs results from Babar/Cleo/Belle/ Focus
- The future of charm physics
 - B-Factories / Cleo-c / Bes III

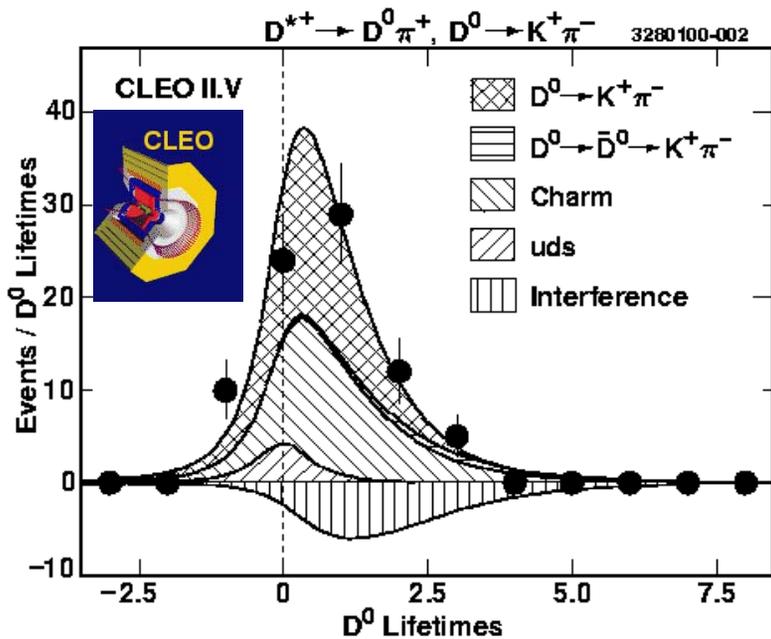
Featuring
results from



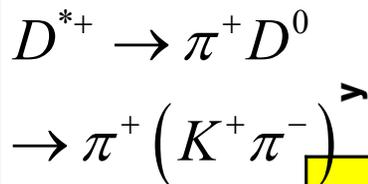
TM & © Nalvana

Apologies for all the important and fascinating results that I had to skip

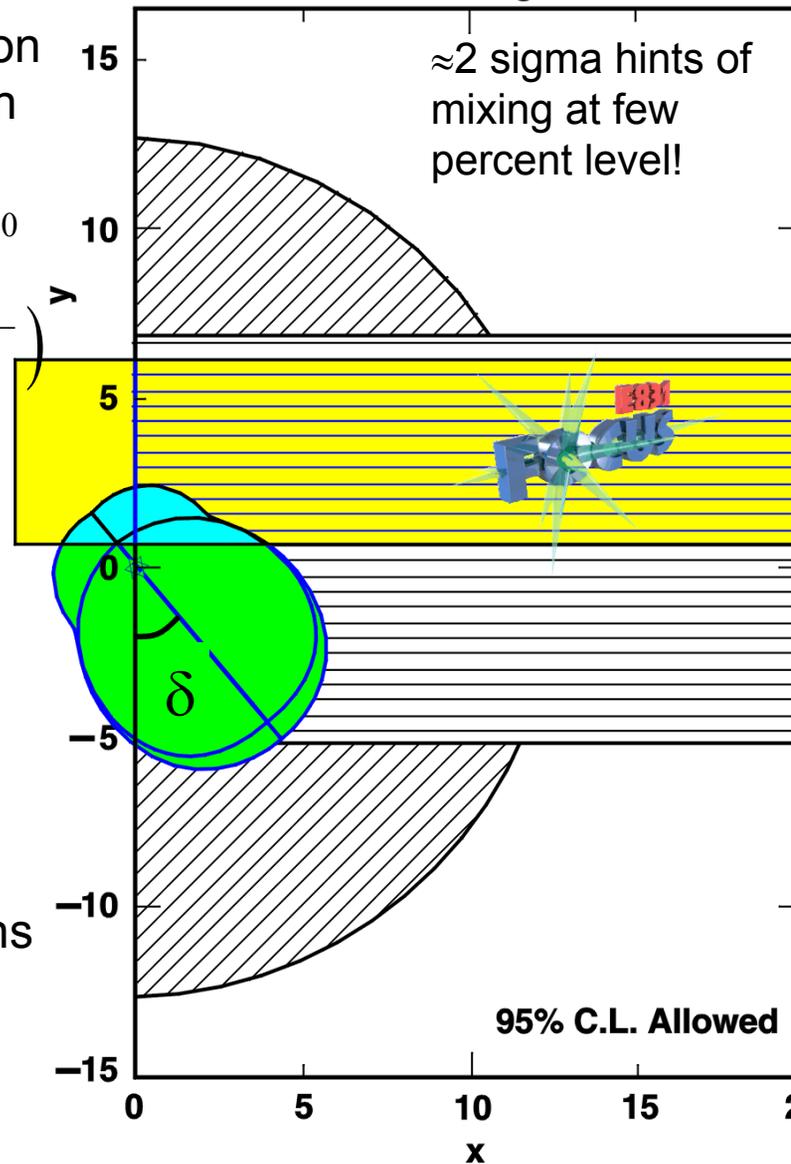
charm mixing circa 2000



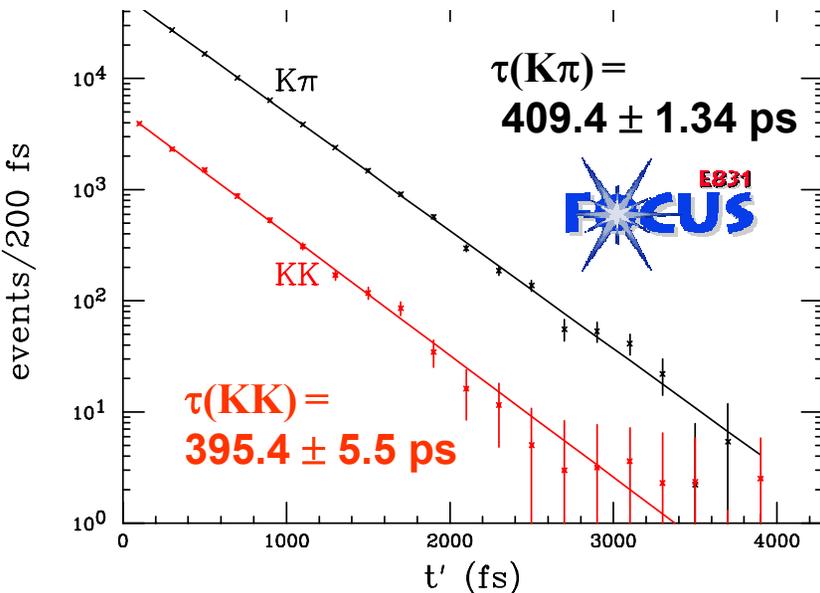
Time evolution
of wrong-sign
 D^* decay



$D^0 - \bar{D}^0$ Mixing Limits



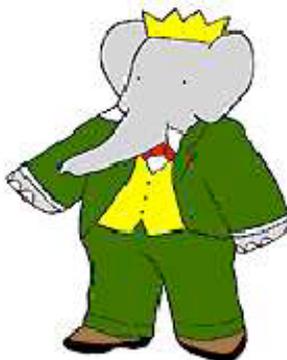
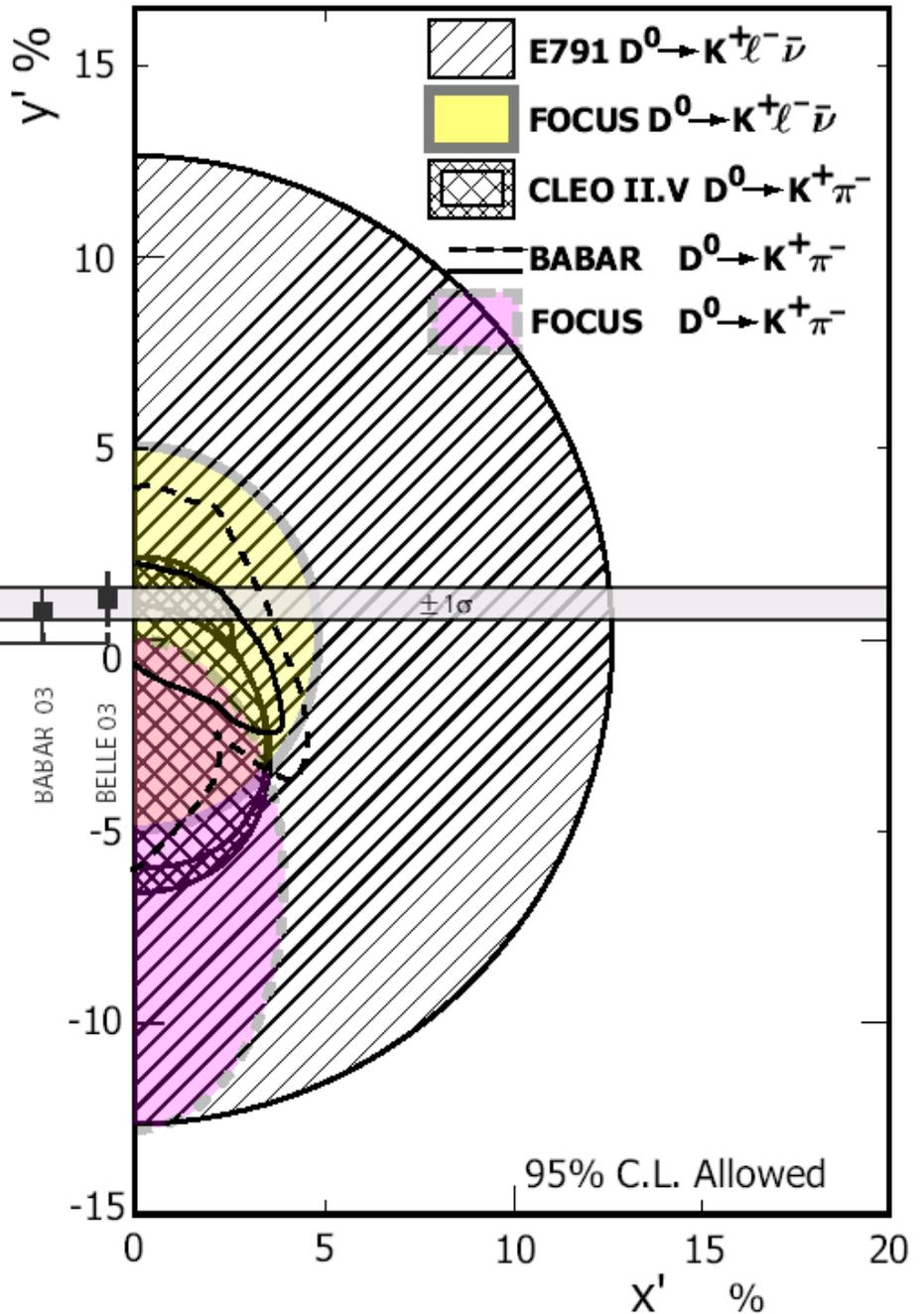
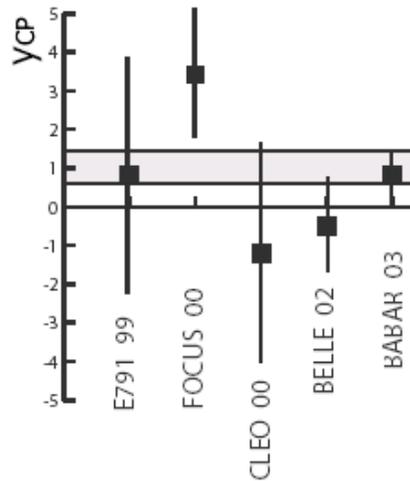
CP lifetime
comparisons



Mixing circa 2003

Things have come a long way since those heady days...

It will be interesting to see if mixing does occur at the percent level.



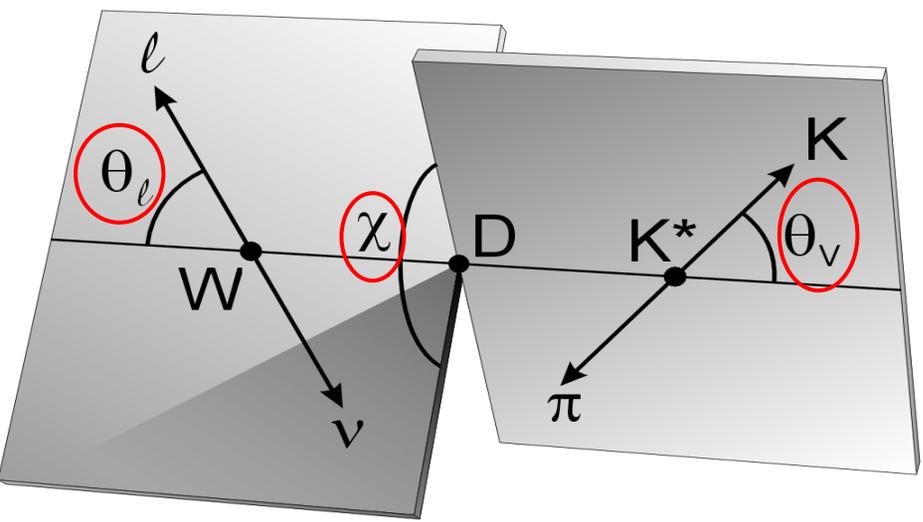
TM & © Nelvana

D → vector μ ν decays

Two amplitude sums over W polarization using D-matrices

$$|A|^2 = \frac{1}{8} (q^2 - m_l^2) \left\{ \begin{array}{l} \text{right-handed } \mu^+ \\ \left(\begin{array}{l} (1 + \cos \theta_l) \sin \theta_V e^{i\chi} H_+ \\ -(1 - \cos \theta_l) \sin \theta_V e^{-i\chi} H_- \\ -2 \sin \theta_l \cos \theta_V H_0 \end{array} \right)^2 \end{array} \right. + \frac{m_\mu^2}{q^2} \left. \left\{ \begin{array}{l} \text{left-handed } \mu^+ \\ \left(\begin{array}{l} \sin \theta_l \sin \theta_V e^{i\chi} H_+ \\ + \sin \theta_l \sin \theta_V e^{-i\chi} H_- \\ + 2 \cos \theta_l \cos \theta_V H_0 \\ + 2 \cos \theta_V H_t \end{array} \right)^2 \end{array} \right. \right\}$$

$H_0(q^2)$, $H_+(q^2)$, $H_-(q^2)$ are helicity-basis form factors computable by LQCD...



Helicity FF are combinations of vector and axial FF

$$A_i(q^2) = \frac{A_i(0)}{1 - q^2/M_A^2}$$

$$V(q^2) = \frac{V(0)}{1 - q^2/M_V^2}$$

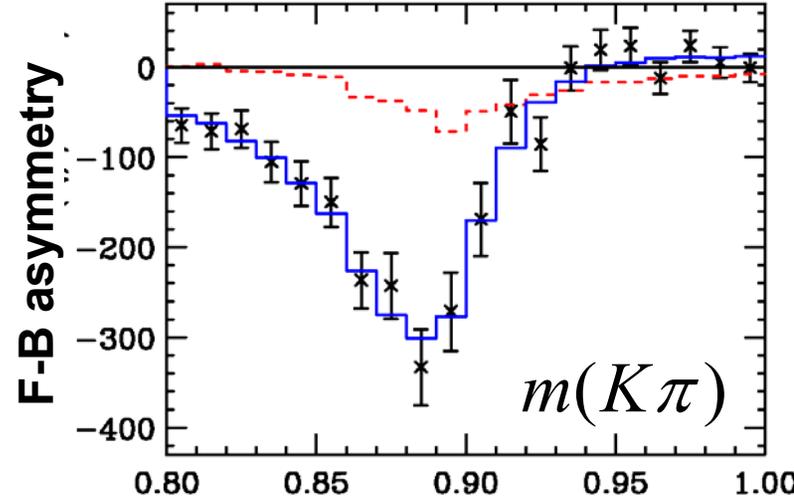
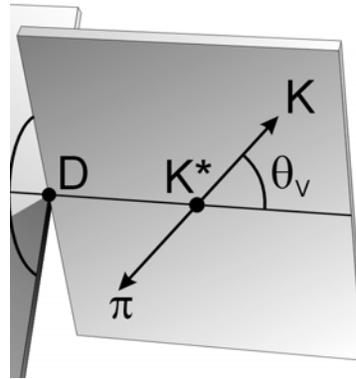
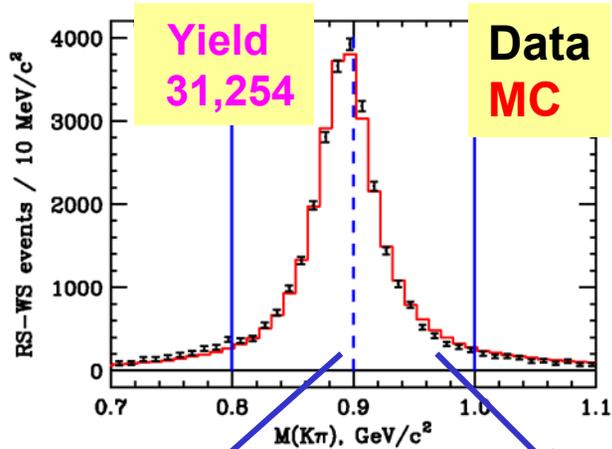
Two numbers parameterize the decay

$$r_v \equiv V(0)/A_1(0)$$

$$r_2 \equiv A_2(0)/A_1(0)$$

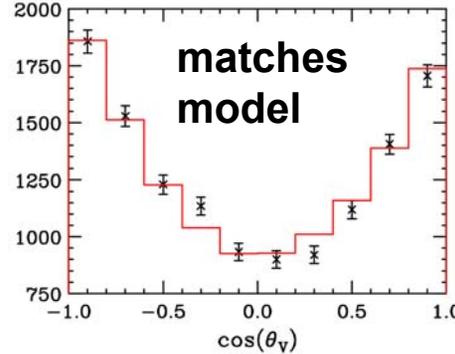
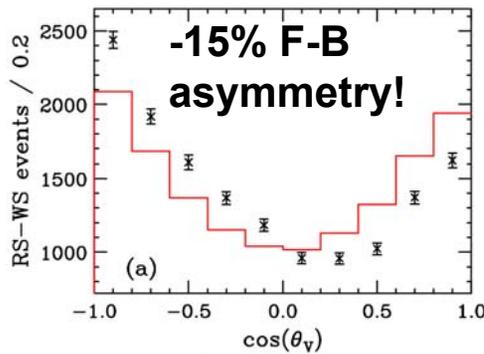
Interference in $D^+ \rightarrow K^* \mu \nu$

Focus "K*" signal



$0.8 < M(K\pi) < 0.9 \text{ GeV}/c^2$

$0.9 < M(K\pi) < 1.0 \text{ GeV}/c^2$



$K^* \mu \nu$ interferes with S-wave $K\pi$ and creates a forward-backward asymmetry in the K^* decay angle with a mass variation due to the varying BW phase

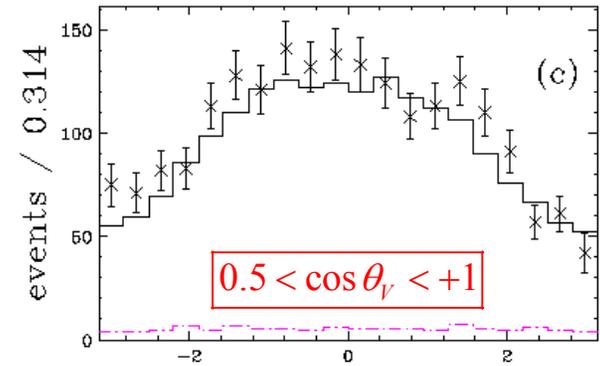
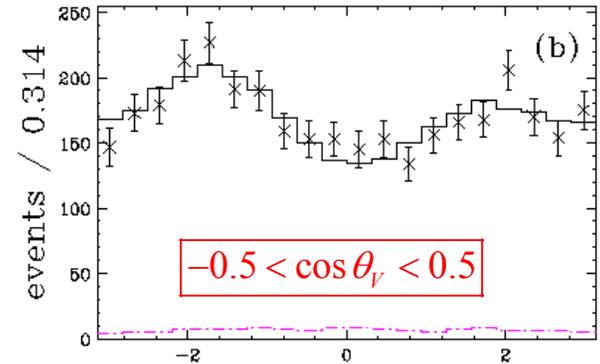
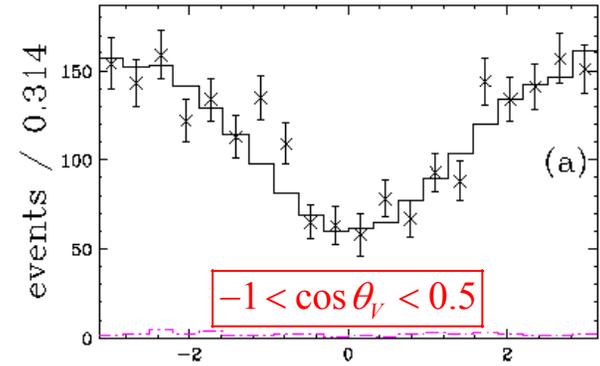
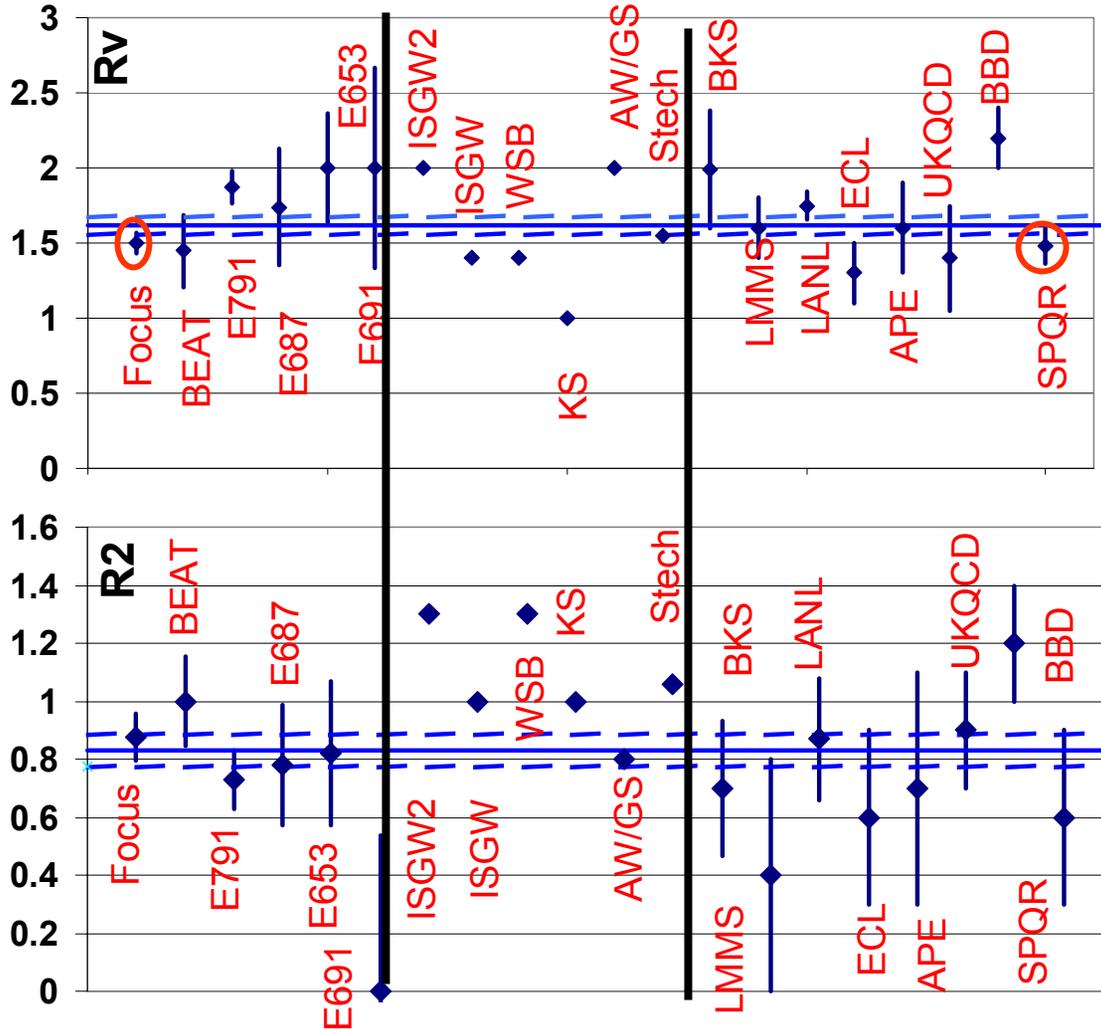
The S-wave amplitude is about 7% of the K^* BW with a 45° relative phase



(2002)

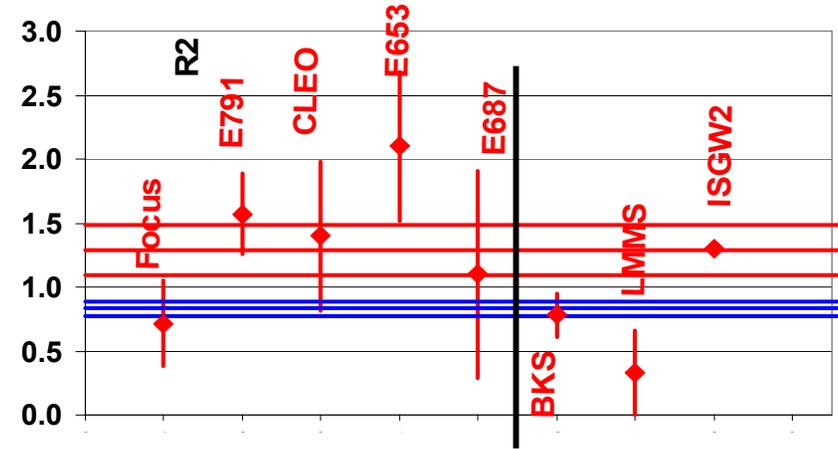
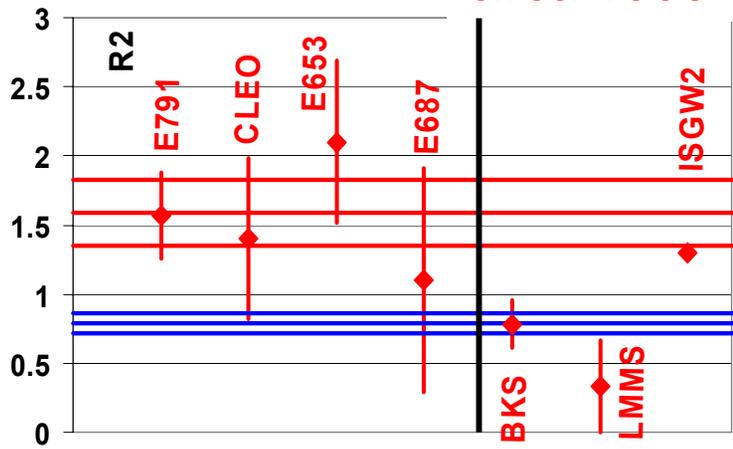
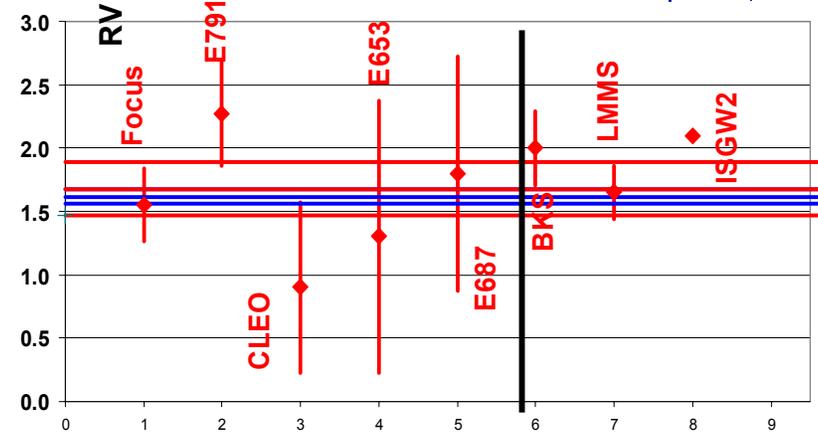
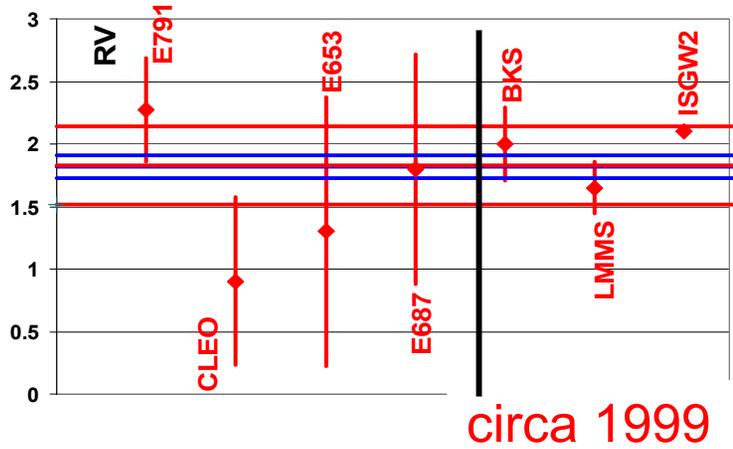
The same relative phase as LASS

$K^*_{\mu\nu}$ form factors



Results are getting very precise and more calculations are needed.

$D_s \rightarrow \phi \mu \nu$ form factors



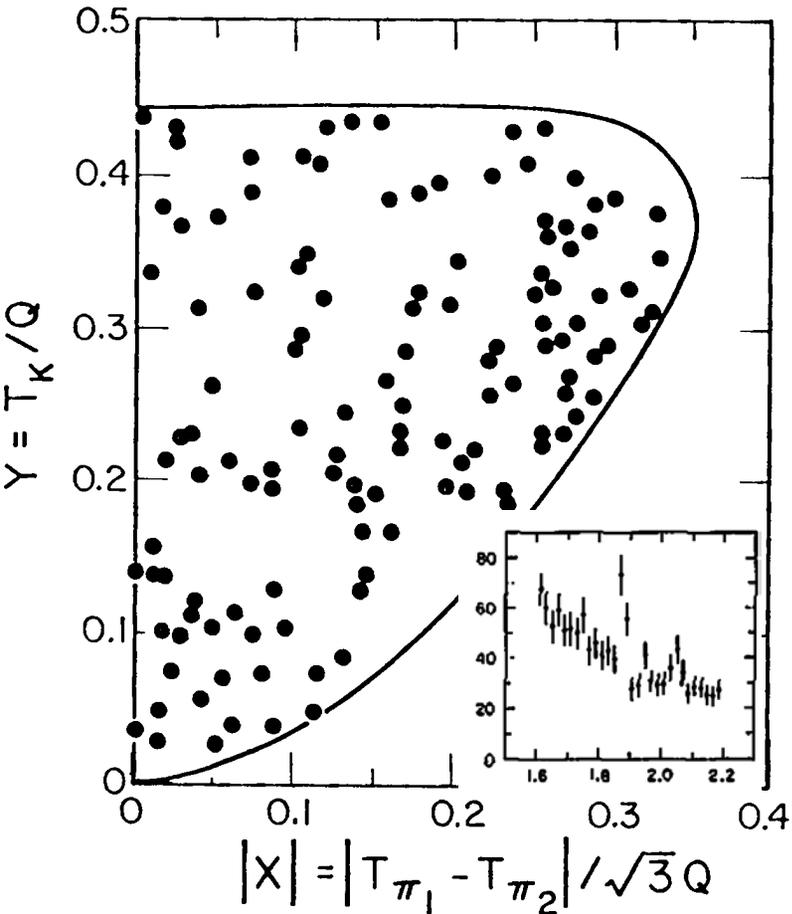
Theoretically the $D_s \rightarrow \phi l \nu$ form factor should be within 10% of $D \rightarrow K^* l \nu$. The r_V values were consistent but r_2 for $D_s \rightarrow \phi l \nu$ was $\approx 2 \otimes$ higher than $D \rightarrow K^* l \nu$.

But the (2004) FOCUS measurement has consistent r_2 values as well!

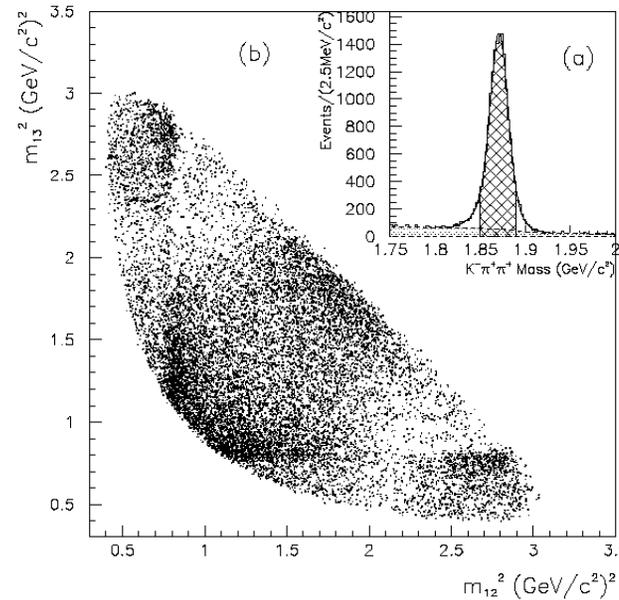
Dalitz plots

E791 (2002)

The first charm Dalitz analysis –
MK1 (1977) $D^+ \rightarrow K\pi\pi$



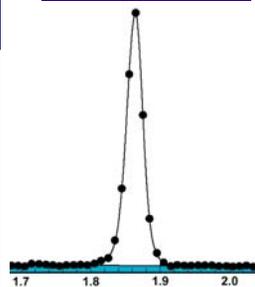
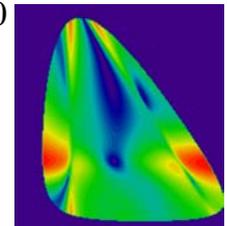
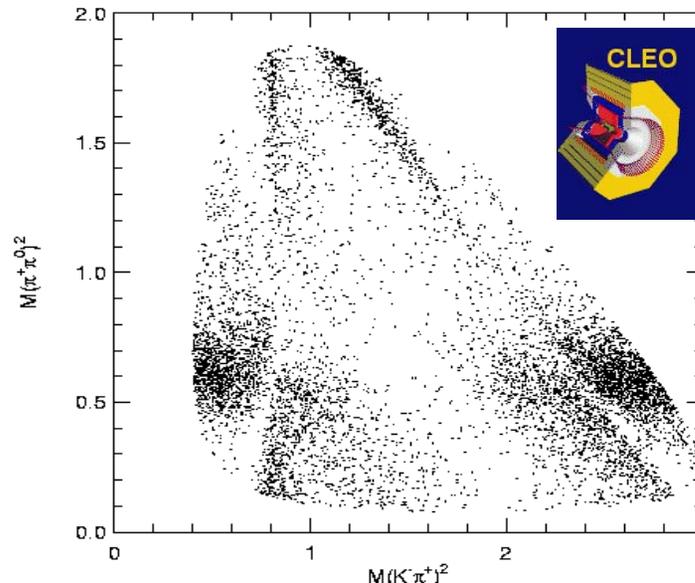
“...consistent with a phase space Dalitz distribution.”



Same state but higher statistics

Lots of structure!

CLEO (2001) $D^0 \rightarrow K^-\pi^+\pi^0$

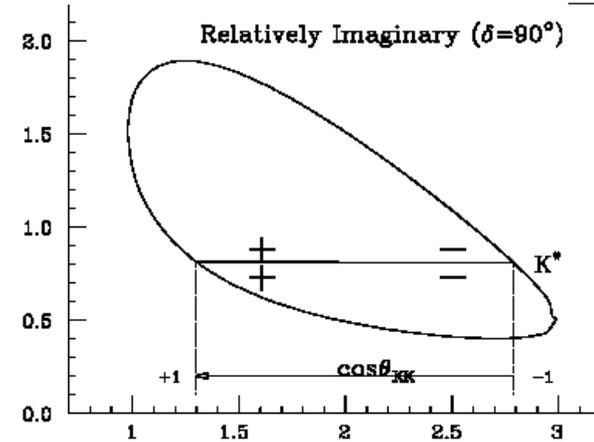
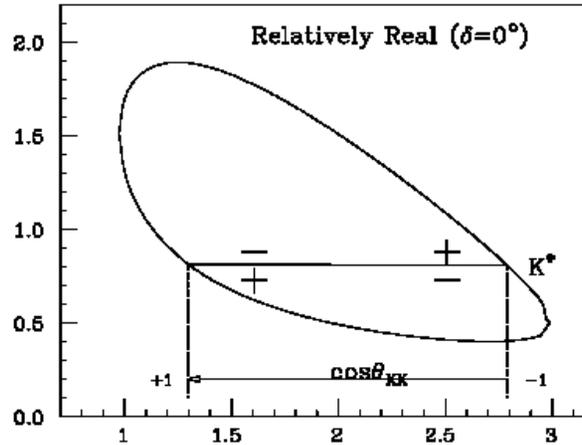
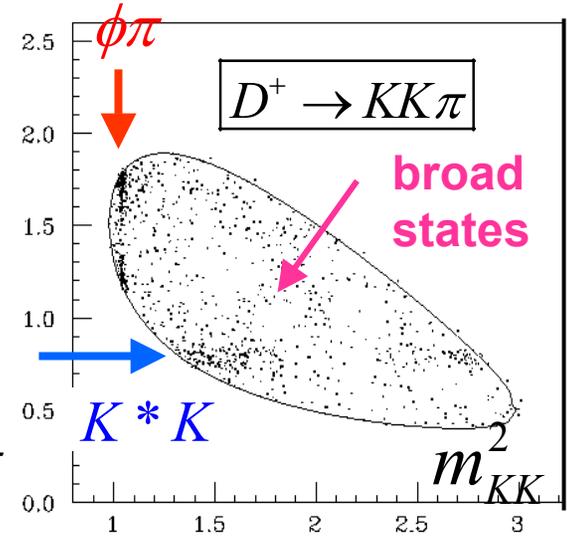
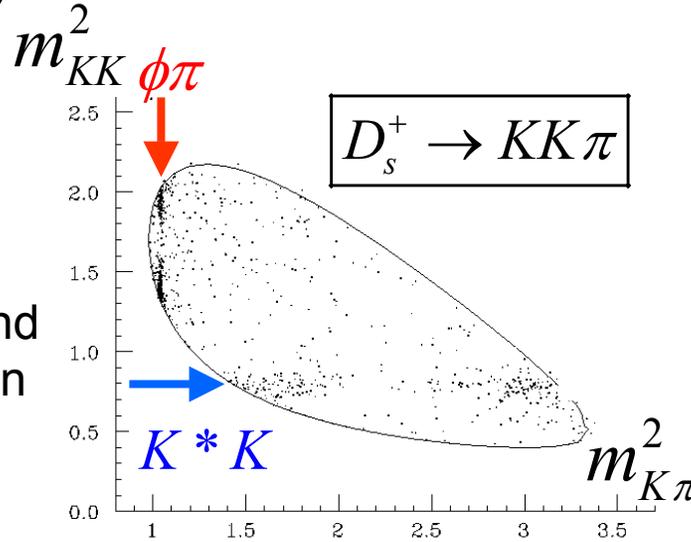


What do you learn from Dalitz plots?

- Bands indicate resonance contributions
- For spinless parents, the number of nodes in the band give you the resonance spin
- Interference pattern gives relative phases and amplitudes

• Look at the ϕ band

• Look at the $D^+ K^*$ band pattern of asymmetry



Isobar model: Add up BW's with angular factors

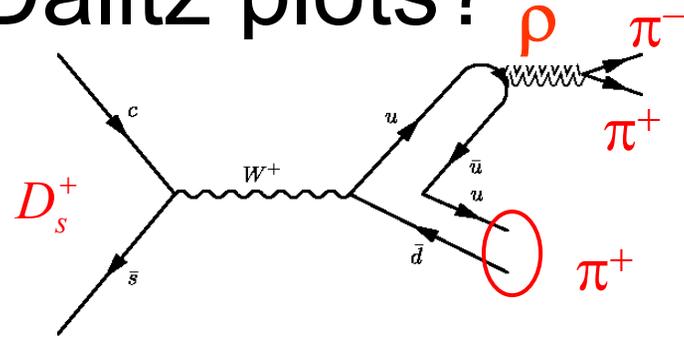
Nearly all charm analyses use the isobar model:

$$\mathcal{M} = \sum_m a_m e^{i\delta_m} A_m$$

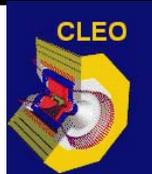
$$A = \frac{|\vec{p}_1|^J |\vec{p}_3|^J P_J(\cos \mathcal{G}_{13}^r)}{m_r^2 - m_{12}^2 - im_r \Gamma_r}$$

Why study charm Dalitz plots?

- A valuable source of phase shifts in entangling γ in $B^\pm \rightarrow K^\pm (K^* K)_D$ (discussed in talk of Paras Naik)
- Probes of charm decay mechanism
Role of direct annihilation in D_s decay



• New probes of charm CP violation



$$\mathcal{M} = a_0 e^{i\delta_0} + \sum_j a_j e^{i(\delta_j + \phi_j)} \left(1 + \frac{b_j}{a_j}\right) \mathcal{A}_j$$

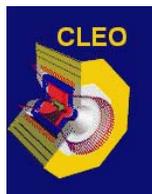
$$\overline{\mathcal{M}} = a_0 e^{i\delta_0} + \sum_j a_j e^{i(\delta_j - \phi_j)} \left(1 - \frac{b_j}{a_j}\right) \mathcal{A}_j$$

SM estimates are 10^{-6} for $(K_s^+ \pi^+ \pi^-)$

Co	Amplitude (b_j/a_j)	Phase (ϕ_j) ($^\circ$)
Cleo $D_s \rightarrow K_s \pi^+ \pi^-$		
$K^*(892)^+ \pi^- \times B(K^*(892)^+ \rightarrow K^0 \pi^+)$	$-0.12 \pm 0.20_{-0.15}^{+0.06+0.10}$	$4 \pm 20_{-3}^{+1+4}$
$\overline{K}^0 \rho^0$	$0.00 \pm 0.02_{-0.06}^{+0.02+0.00}$	$-3 \pm 16_{-2}^{+0+6}$
$\overline{K}^0 \omega \times B(\omega \rightarrow \pi^+ \pi^-)$	$-0.09 \pm 0.10_{-0.00}^{+0.07+0.01}$	$-8 \pm 17_{-2}^{+2+6}$
$K^*(892)^- \pi^+ \times B(K^*(892)^- \rightarrow \overline{K}^0 \pi^-)$	$0.00 \pm 0.02_{-0.06}^{+0.01+0.01}$	$-5 \pm 15_{-1}^{+0+5}$
$\overline{K}^0 f_0(980) \times B(f_0(980) \rightarrow \pi^+ \pi^-)$	$-0.03 \pm 0.05_{-0.06}^{+0.04+0.04}$	$7 \pm 15_{-1}^{+1+4}$
$\overline{K}^0 f_2(1270) \times B(f_2(1270) \rightarrow \pi^+ \pi^-)$	$0.15 \pm 0.23_{-0.19}^{+0.14+0.13}$	$21 \pm 18_{-15}^{+2+28}$
$\overline{K}^0 f_0(1370) \times B(f_0(1370) \rightarrow \pi^+ \pi^-)$	$0.08 \pm 0.05_{-0.15}^{+0.05+0.15}$	$7 \pm 14_{-3}^{+1+12}$
$K_0^*(1430)^- \pi^+ \times B(K_0^*(1430)^- \rightarrow \overline{K}^0 \pi^-)$	$-0.02 \pm 0.05_{-0.06}^{+0.07+0.06}$	$-5 \pm 16_{-3}^{+1+8}$
$K_2^*(1430)^- \pi^+ \times B(K_2^*(1430)^- \rightarrow \overline{K}^0 \pi^-)$	$-0.06 \pm 0.11_{-0.11}^{+0.03+0.11}$	$1 \pm 16_{-2}^{+2+10}$
$K^*(1680)^- \pi^+ \times B(K^*(1680)^- \rightarrow \overline{K}^0 \pi^-)$	$-0.20 \pm 0.09_{-0.04}^{+0.11+0.12}$	$-6 \pm 16_{-0}^{+17+14}$

• Unique mixing probes

Time dependent Dalitz fit such as $D \rightarrow K_s \pi^+ \pi^-$ (David Asner)



$$|M|^2 = |A|^2 \exp(-\Gamma_1 t) + |B|^2 \exp(-\Gamma_2 t) +$$

$$2 |A| |B| \exp(-\bar{\Gamma} t) \left\{ \cos(\delta) \cos(x \bar{\Gamma} t) - \boxed{\sin(\delta) \sin(x \bar{\Gamma} t)} \right\}$$

A includes $\{K_s f_0(980), K_s f_0(1370), \dots\}$

B includes $\{K_s \rho(770), K_s \rho(1450), \dots\}$

and $K^* \pi$ contributes to both A and B

Sensitive to sign of x

Although used frequently, the isobar model has problems ...

The isobar amplitude is a sum of BW

$$M = \sum a_i e^{i\delta_i} BW \otimes \text{angfactors} \approx \sum T_i$$

Unitary (single channel for simplicity)

$$S = 1 + 2iT; \quad S^* S = 1 \rightarrow \text{Im} T = |T|^2$$

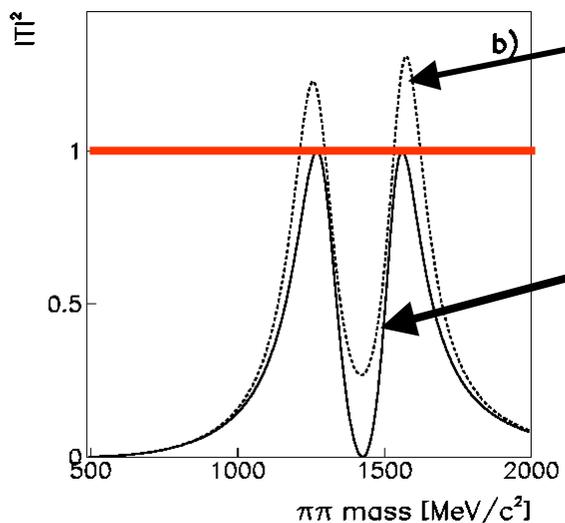
Unitarity can be achieved with real K

$$T = \frac{K}{1 - iK} \rightarrow \text{Im} T = \frac{K^2}{1^2 + K^2} = |T|^2$$

$$K = \frac{m\Gamma}{m^2 - s} \rightarrow T = \frac{m\Gamma}{m^2 - s - im\Gamma}$$

A real pole in K gives a BW

The K and T descriptions are equivalent for a single pole but what happens when several poles contribute??

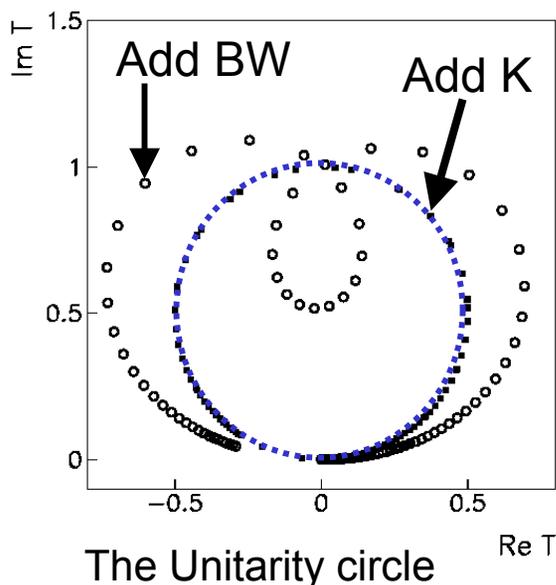


Add two BW ala Isobar model

Add two K matrices

Adding BW violates unitarity

Adding K matrices respects unitarity



When broad resonances overlap you just can't add Breit-Wigners and respect QM!

FOCUS D $\rightarrow \pi^+ \pi^+ \pi^-$ analysis

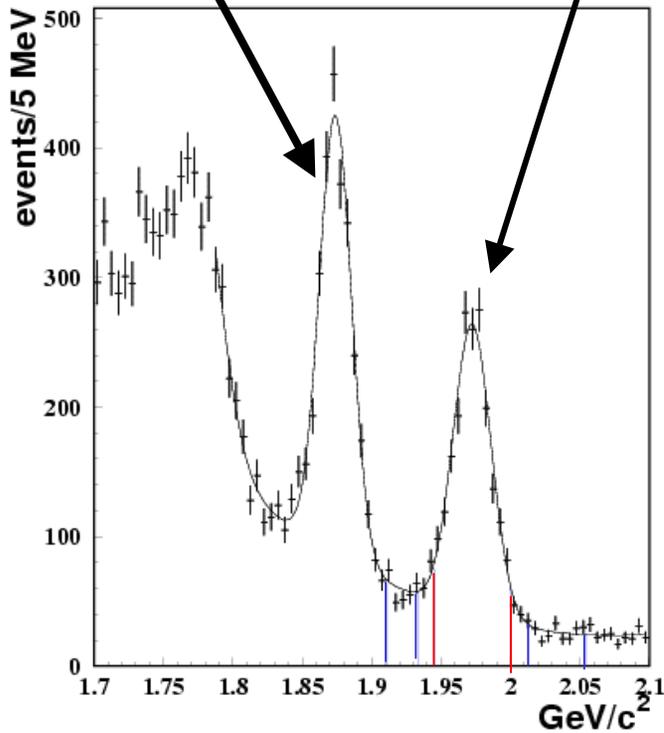
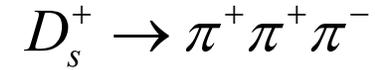


Yield $D^+ = 1527 \pm 51$

S/N $D^+ = 3.64$

Yield $D_s^+ = 1475 \pm 50$

S/N $D_s^+ = 3.41$

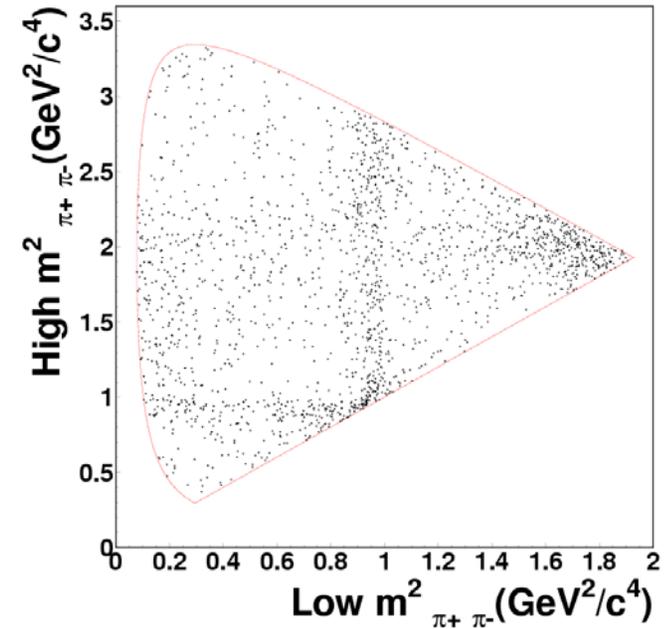


Observe:

• $f_0(980)$

• $f_2(1270)$

• $f_0(1500)$

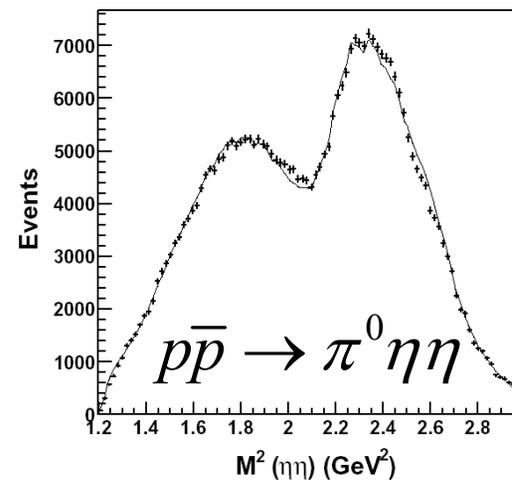
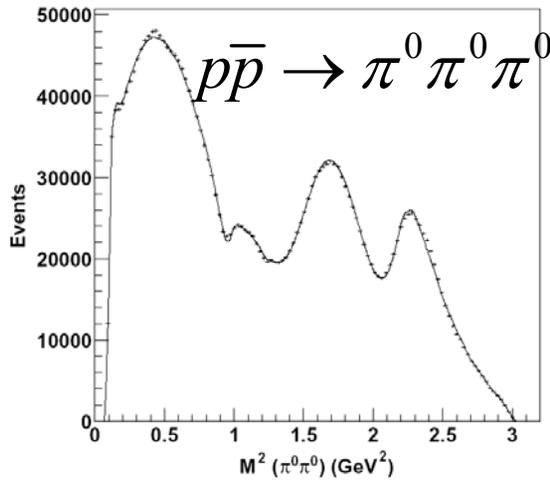


Several broad and overlapping resonances contribute

Can they fit it using K- matrix based on fits to other data??

“K-matrix analysis of the 00+-wave in the mass region below 1900 MeV”

V.V Anisovich and A.V.Sarantsev Eur.Phys.J.A16 (2003) 229



GAMS
GAMS
BNL
CERN-Munich
Crystal Barrel
Crystal Barrel
Crystal Barrel
Crystal Barrel
E852

$\pi p \rightarrow \pi^0 \pi^0 n, \eta \eta n, \eta \eta' n, |t| < 0.2 \text{ (GeV}^2/c^2)$
 $\pi p \rightarrow \pi^0 \pi^0 n, 0.30 < |t| < 1.0 \text{ (GeV}^2/c^2)$
 $\pi p \rightarrow K \bar{K} n$
 $\pi^+ \pi^- \rightarrow \pi^+ \pi^-$
 $\bar{p} p \rightarrow \pi^0 \pi^0 \pi^0, \pi^0 \pi^0 \eta, \pi^0 \eta \eta$
 $\bar{p} p \rightarrow \pi^0 \pi^0 \pi^0, \pi^0 \pi^0 \eta$
 $\bar{p} p \rightarrow \pi^+ \pi^- \pi^0, K^+ K^- \pi^0, K_s^+ K_s^- \pi^0, K^+ K_s^- \pi^-$
 $\bar{p} p \rightarrow \pi^0 \pi^0 \pi^0, \pi^+ \pi^- \pi^+, K_s^+ K_s^- \pi^0, K_s^+ K_s^- \pi^-$
 $\pi p \rightarrow \pi^0 \pi^0 n, 0 < |t| < 1.5 \text{ (GeV}^2/c^2)$

	$\pi\pi$	$K\bar{K}$	4π	$\eta\eta$	$\eta\eta'$
<i>Poles</i>	$g_{\pi\pi}$	$g_{K\bar{K}}$	$g_{4\pi}$	$g_{\eta\eta}$	$g_{\eta\eta'}$
	0.65100	-0.52523	0	-0.38878	-0.36397
	1.20720	0.55427	0	0.38705	0.29448
	1.56122	0.23591	0.62605	0.18409	0.18923
	1.21257	0.39642	0.97644	0.19746	0.00357
	1.81746	-0.17915	-0.90100	-0.00931	0.20689
s_0^{scatt}	f_{11}^{scatt}	f_{12}^{scatt}	f_{13}^{scatt}	f_{14}^{scatt}	f_{15}^{scatt}
	-3.30564	0.16583	-0.19840	0.32808	0.31193

An impressive amount of data is well described in terms of 5 poles

$$K_{ij}(s) = \left(\sum_{\alpha} \frac{g_i^{(\alpha)} g_j^{(\alpha)}}{m_{\alpha}^2 - s} + f_{ij}^{scatt} \frac{1 - s_0^{scatt}}{s - s_0^{scatt}} \right)$$

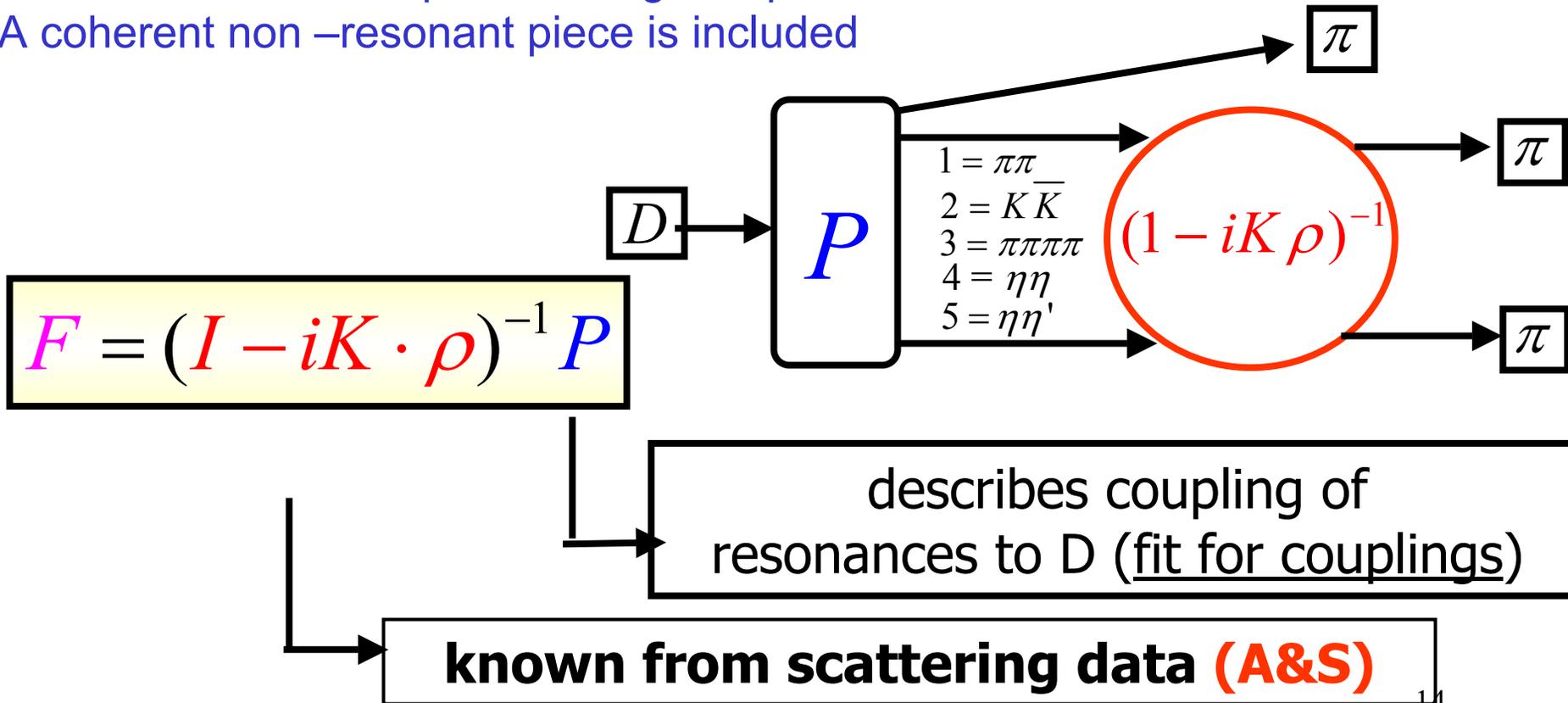
K-matrix picture

- The Focus amplitude was written as a sum

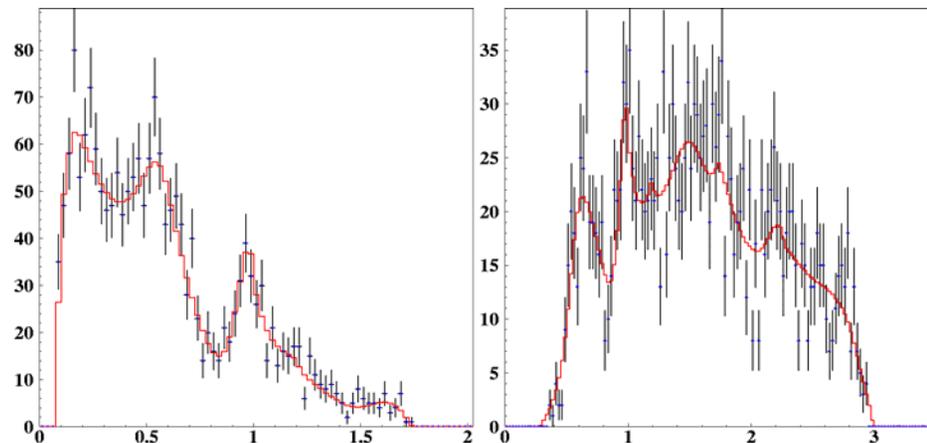
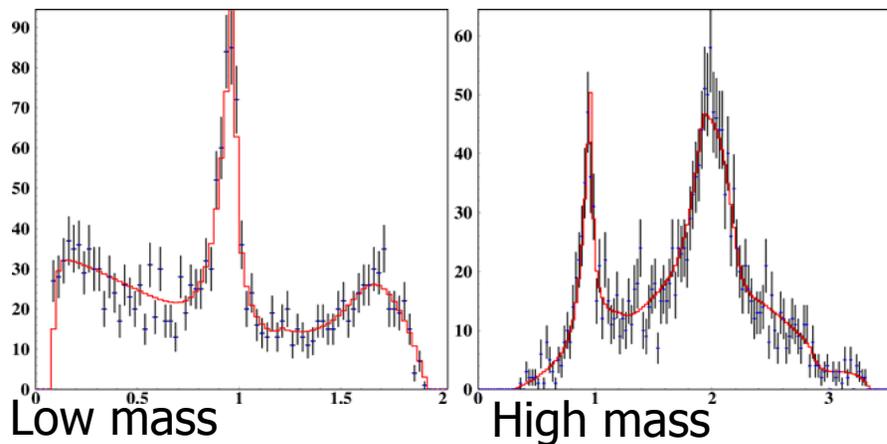
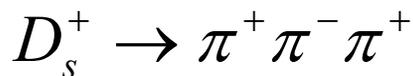
$$A(D) = a_0 e^{i\delta_0} + F + \sum_i^{J>0} a_i e^{i\delta_i} BW$$

vector and
tensor
contributions

- F term models S-wave using five virtual states $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$, 4π
- An isobar BW sum represents higher spin resonances
- A coherent non-resonant piece is included



First K matrix fits to charm Dalitz plots



s-wave dominates

decay channel	fit fractions (%)	phase (deg)
$(S - wave)\pi^+$	$87.04 \pm 5.60 \pm 4.17 \pm 1.34$	$0(\text{fixed})$
$f_2(1270)\pi^+$	$9.74 \pm 4.49 \pm 2.63 \pm 1.32$	$168.0 \pm 18.7 \pm 2.5 \pm 21.7$
$\rho^0(1450)\pi^+$	$6.56 \pm 3.43 \pm 3.31 \pm 2.90$	$234.9 \pm 19.5 \pm 13.3 \pm 24.9$

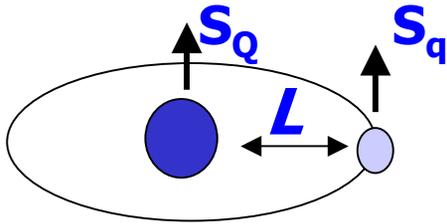
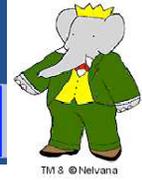
decay channel	fit fractions (%)	phase (deg)
$(S - wave)\pi^+$	$56.00 \pm 3.24 \pm 2.08$	$0(\text{fixed})$
$f_2(1275)\pi^+$	$11.74 \pm 1.90 \pm 0.23$	$-47.5 \pm 18.7 \pm 11.7$
$\rho^0(770)\pi^+$	$30.82 \pm 3.14 \pm 2.29$	$-139.4 \pm 16.5 \pm 9.9$

Reasonable fits with no retuning of the A&S K-matrix and no need to invoke new resonances (such as $\sigma(400)$) and no NR term



L=1 charm mesons

One of few “good” applications of HQET to charm

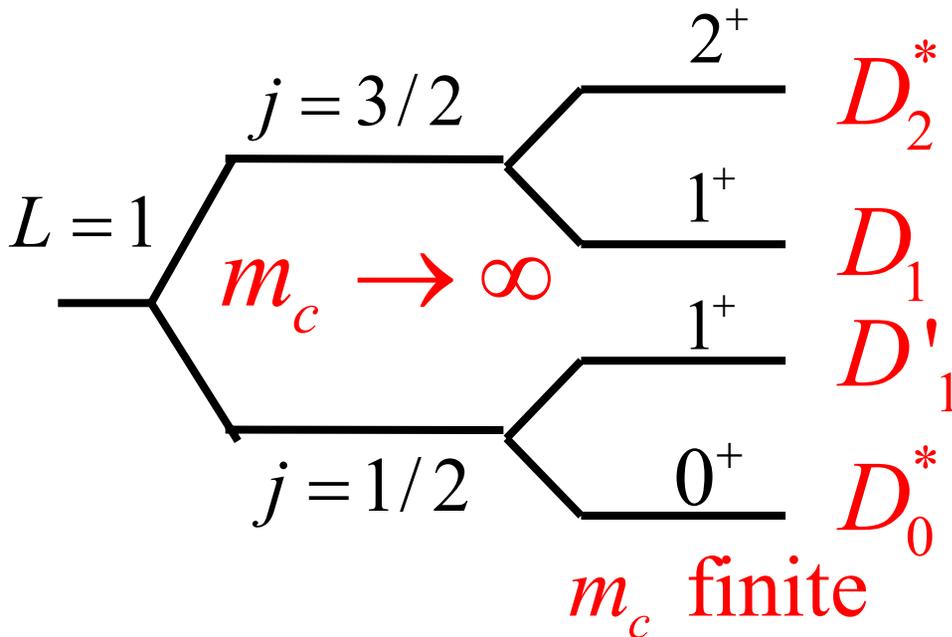


Q spin decouples leaving leaving $j = L \oplus S_q$ as a “good” Q-num
 $(L=1) \otimes (S_q=1/2) \rightarrow j=3/2 \text{ and } 1/2$

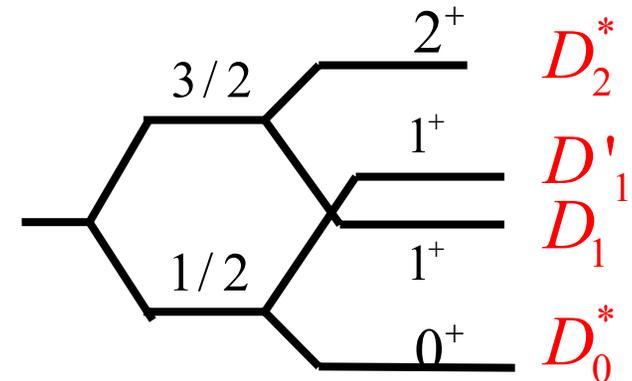
The S_q acts as a perturbation and creates additional splitting

Splitting might look like this

Adding $S_q=1/2$



Of course alternative splitting patterns are possible...



The axial states might switch

Pion transitions to D and D*

Transitions are parity + \rightarrow -

Hence L_π is even

D and D* are $j_q = 1/2$

$$\frac{3}{2} \rightarrow \frac{1}{2} \otimes L_\pi \rightarrow \text{D-wave (narrow)}$$

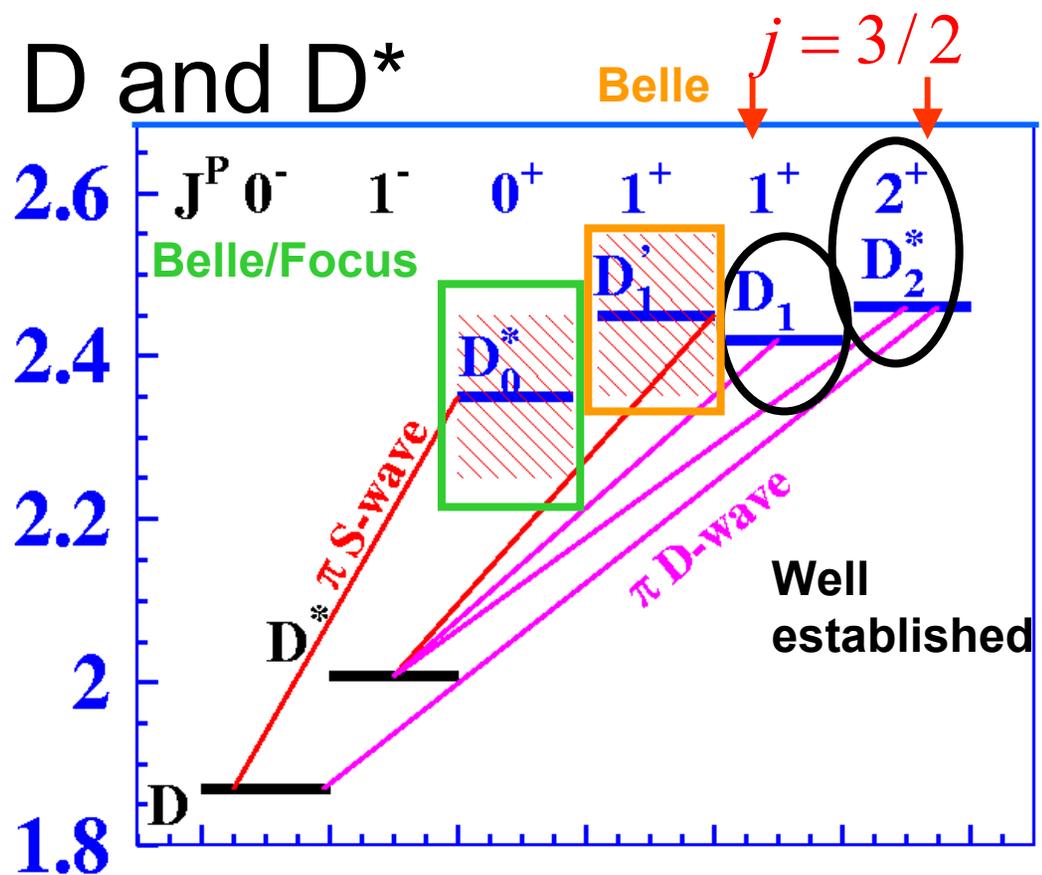
$$\Gamma = 20 \rightarrow 40 \text{ MeV}$$

$$\frac{1}{2} \rightarrow \frac{1}{2} \otimes L_\pi \rightarrow \text{S-wave (broad)}$$

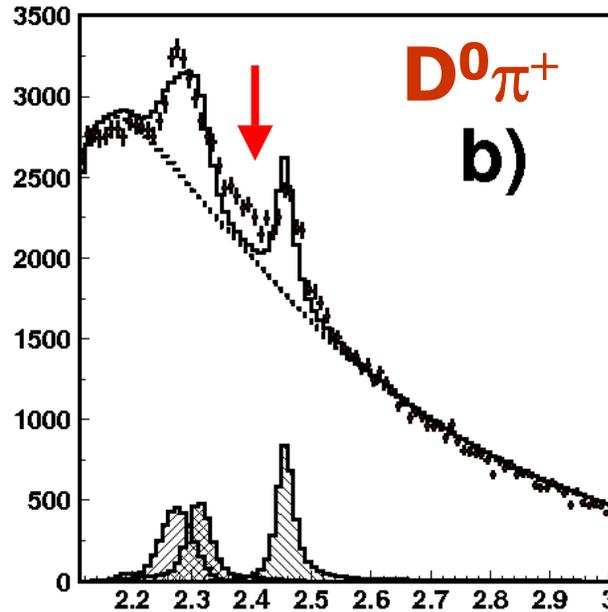
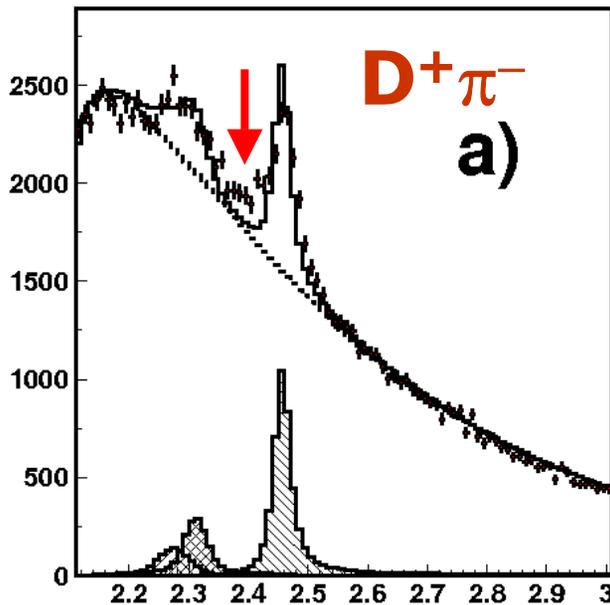
$$\Gamma \geq 200 \text{ MeV}$$

Expect 12 such L=1 states

$$4 \otimes \bar{c}u \quad \bar{c}d \quad c\bar{s}$$



Analysis of mass $D\pi$ spectra

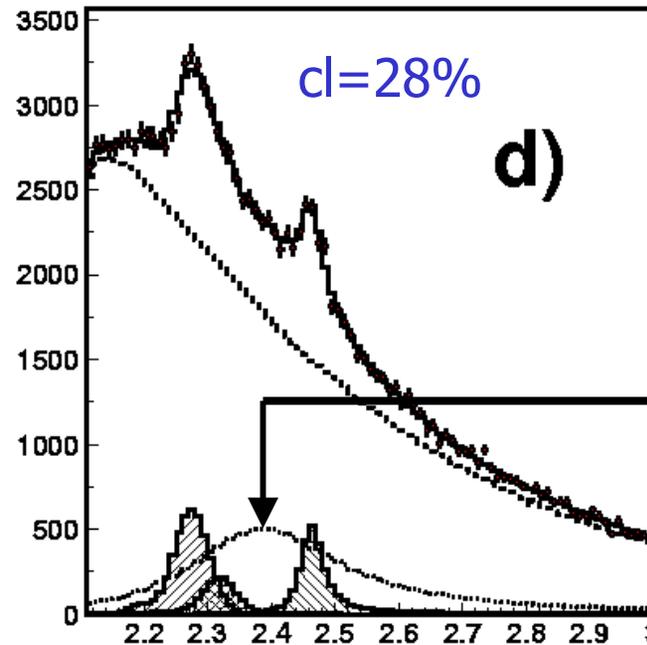
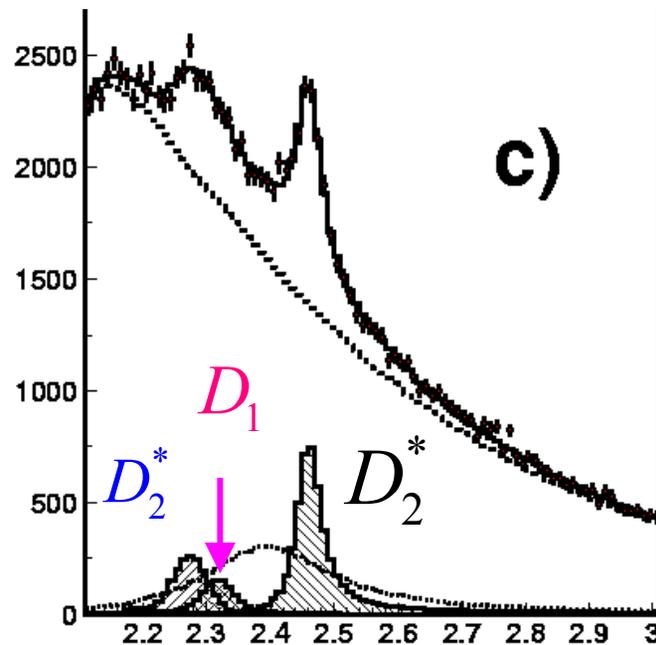


A **very bad fit** with just D_2^* (2460) + D_1 (2420) and feed-downs

$$D_2^* \rightarrow D\pi$$

$$D_2^* \rightarrow D^*\pi$$

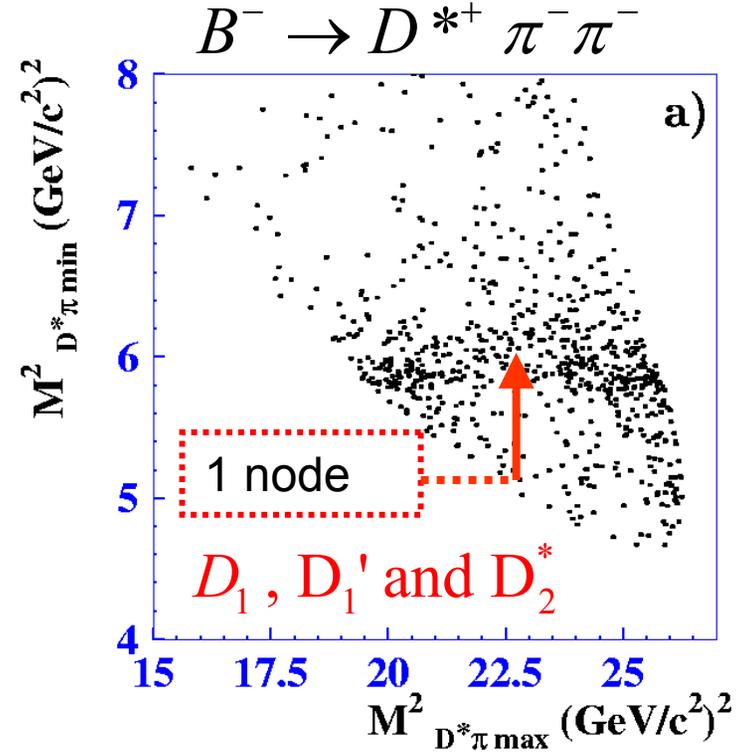
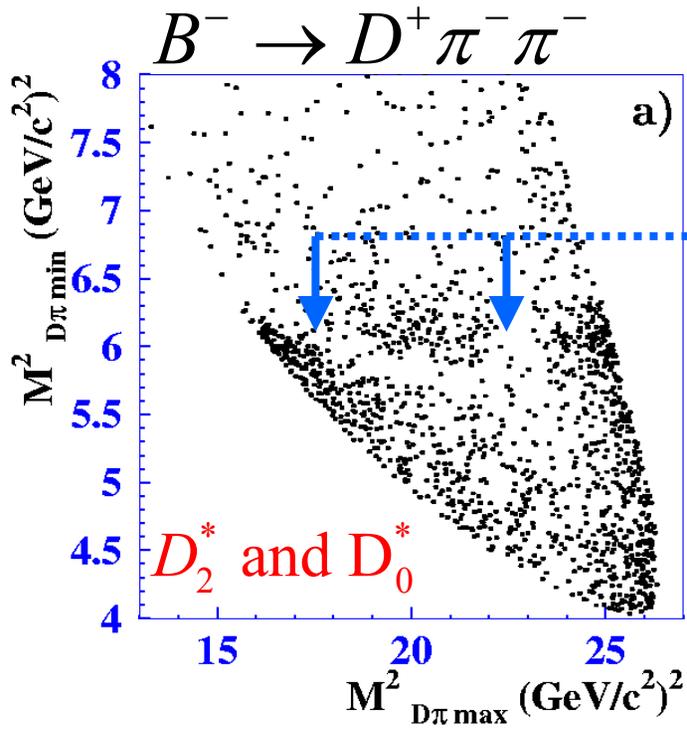
$$\rightarrow (D\pi)\gamma$$



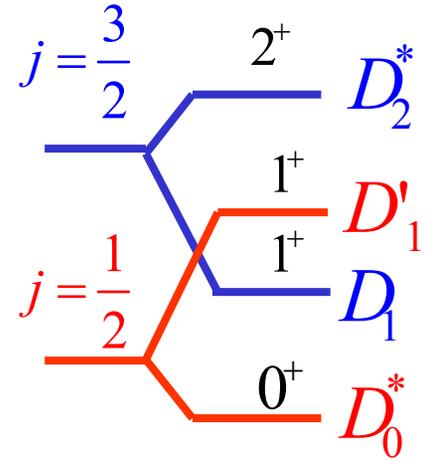
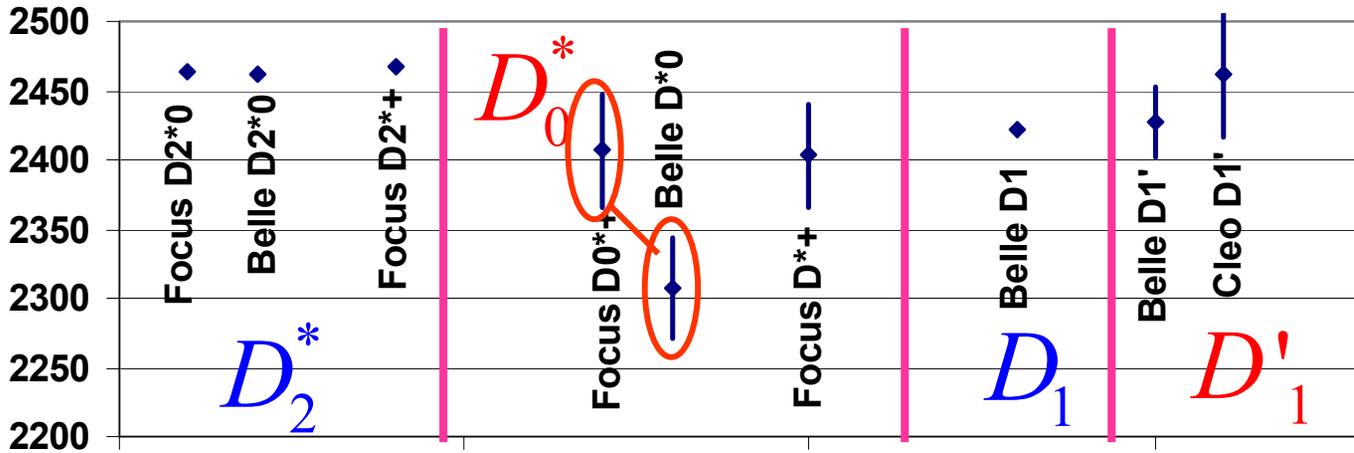
Fit improves dramatically with inclusion of a broad S-wave state

Might be $D_0^* \rightarrow D\pi$ or $D_1' \rightarrow D^*\pi \rightarrow (D\pi)\gamma$
or both

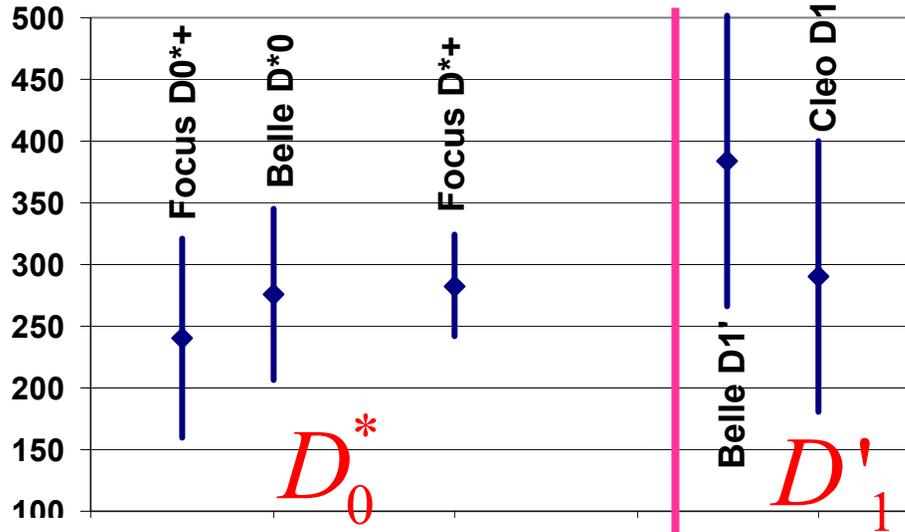
Belle measured the spin 1 states as well



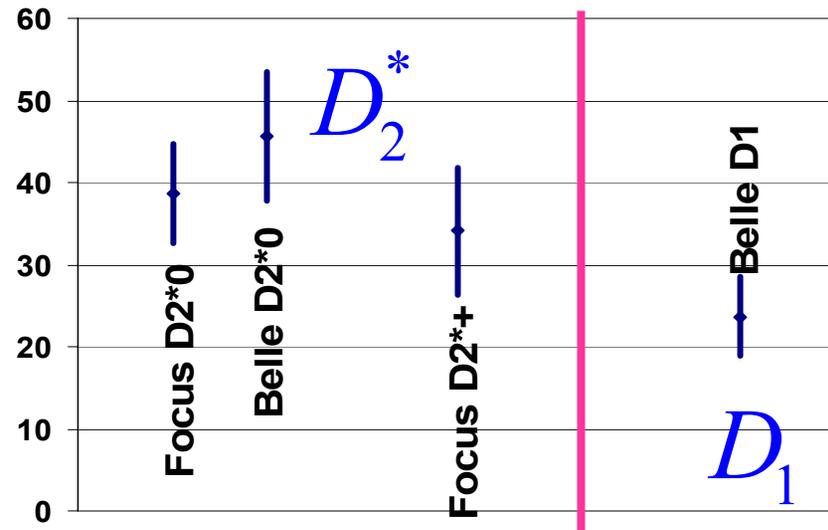
L=1 cu and cd results



Would be great to resolve this



Broad widths



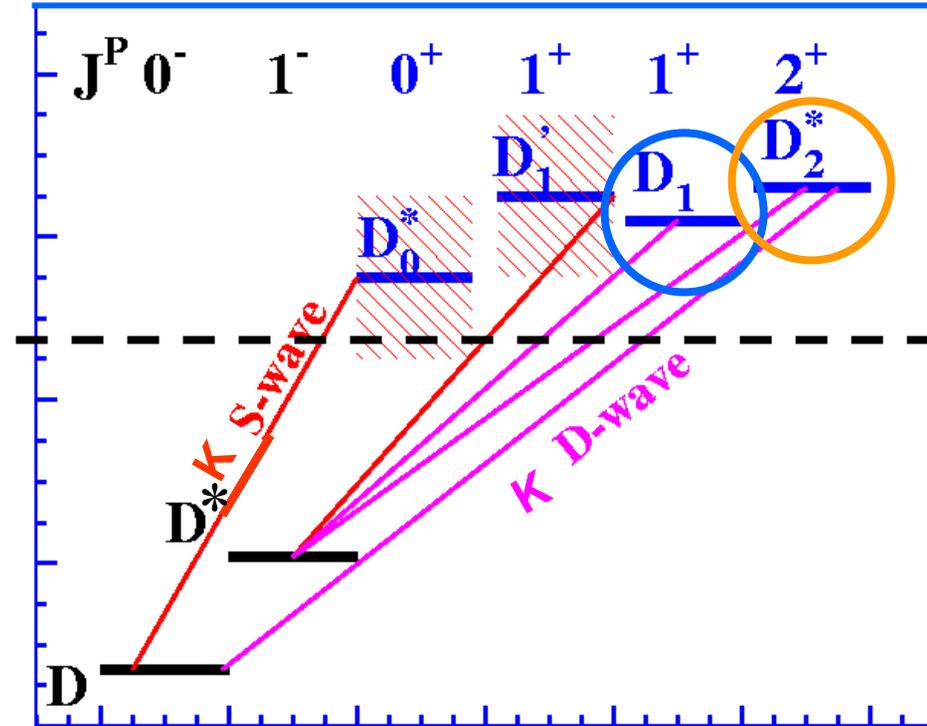
Narrow widths

So on to $L=1$ cs states.

- Naively expects the same spectroscopy and same pattern of decays to D and D* pion→kaon

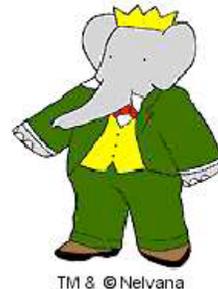
- Two narrow states were seen in DK and D*K transitions. Presumably the $j_q=3/2$ spin 1 and spin 2 states.

- You then expect to see two broad states which were expected to lie above the kaon threshold and decay via s-wave

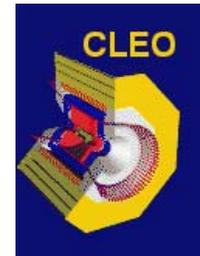


Are two broad states there???

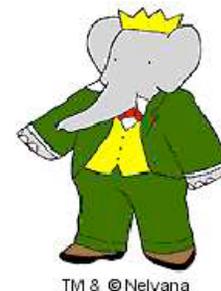
Ask the experts?



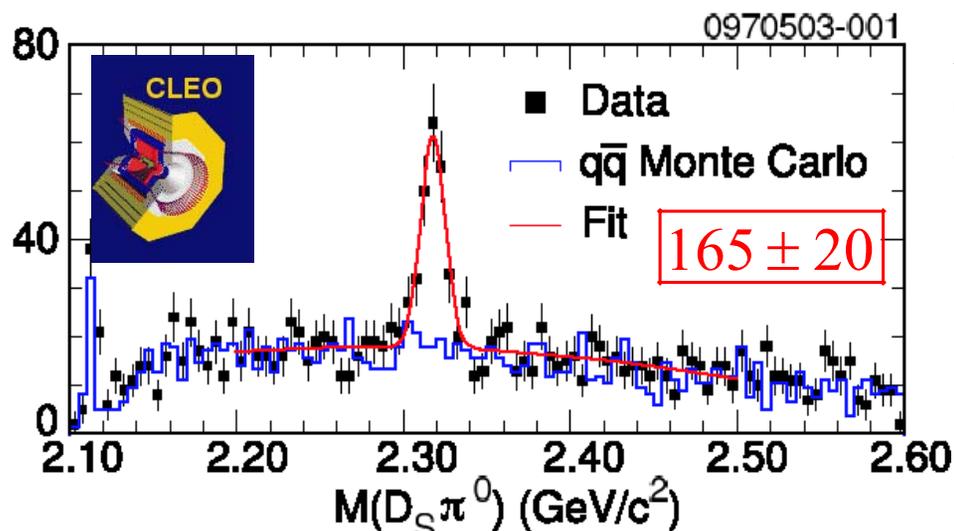
TM & © Nelvana



New excited Ds states



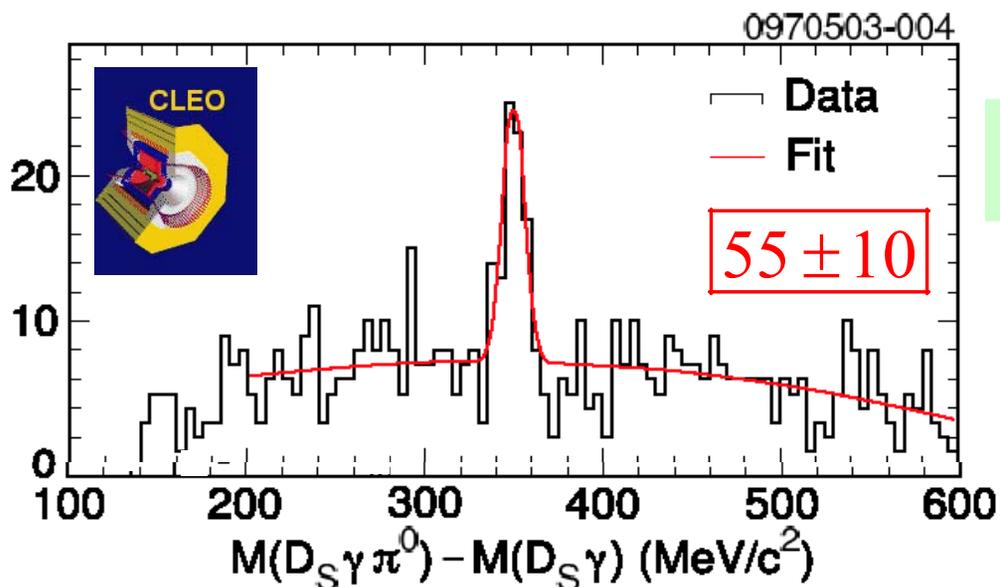
TM & © Nelvana



A new state D_{sJ}^* (2317) was observed by BaBar but with a very small width ($\Gamma < 7 \text{ MeV}$)

This was confirmed by CLEO and a new state was discovered D_{sJ} (2463) again with a very small width

These could be the $j_q=1/2$ states but are unexpectedly low in mass.



These states would be narrow if they lie below D K threshold and must decay into D_s

Decays to D_s violate strong symmetries

$D_{sJ}(2317) \rightarrow D_s \pi$ violates Iso

$D_{sJ}(2317) \rightarrow D_s \pi^+ \pi^-$ violates parity

$D_{sJ}(2317) \rightarrow D_s \gamma$ violates \vec{J}

What are the D_{sJ}^* (2317) and D_{sJ} (2463)?

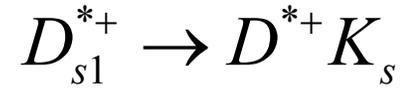
- States consistent with $J^P = 0^+ \text{ \& } 1^+$ given pattern of observed and absent decay modes. They could be the $j=1/2$ cs, But why the low mass?
- DK molecule? (Barnes/Close/Lipkin)
- 4 quark state
- $j=1/2$ (1^+ , 0^+) states are chiral partners to the (1^- 0^-) or (D_s^* D_s) (Bardeen/Eichten/Hill)

Bardeen/Eichten/Hill predict a Goldberger-Trieman-like splitting for 1^\pm and 0^\pm states which works for Ds_1 , Ds_0^*

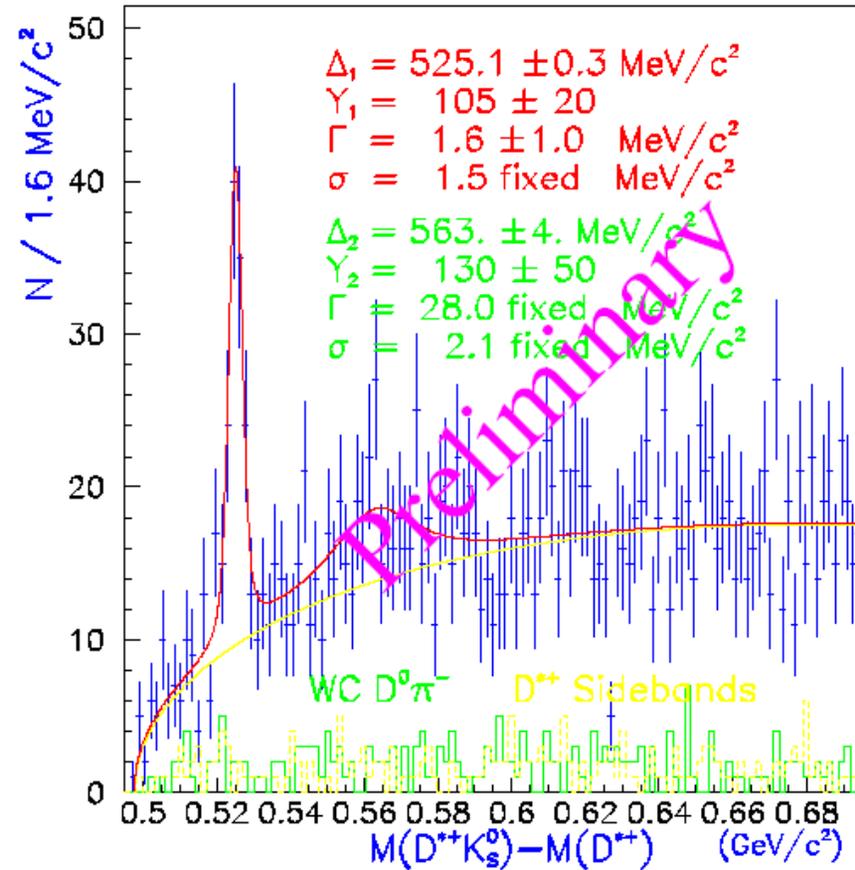
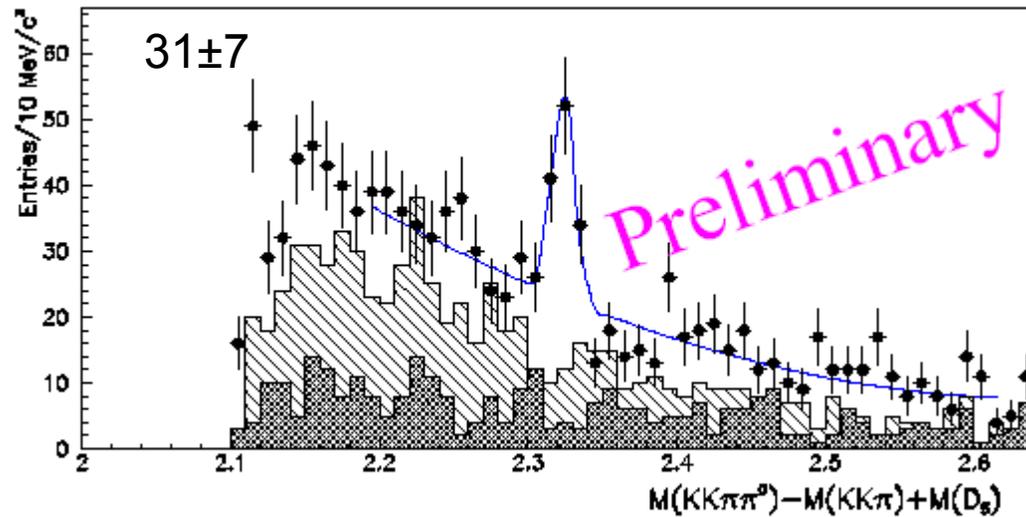
$$\begin{aligned} \delta(\Delta m) &= (351.2 \pm 1.7) - (350.0 \pm 1.2) \\ &\quad 1^+ - 1^- \quad \quad 0^+ - 0^- \\ &= 1.2 \pm 2.1 \text{ MeV} \end{aligned}$$

But: $1^+ - 1^- \approx 0^+ - 0^- \approx 430 \text{ MeV}$ for cu and cd based on Belle measurements

Some preliminary cs results from Focus



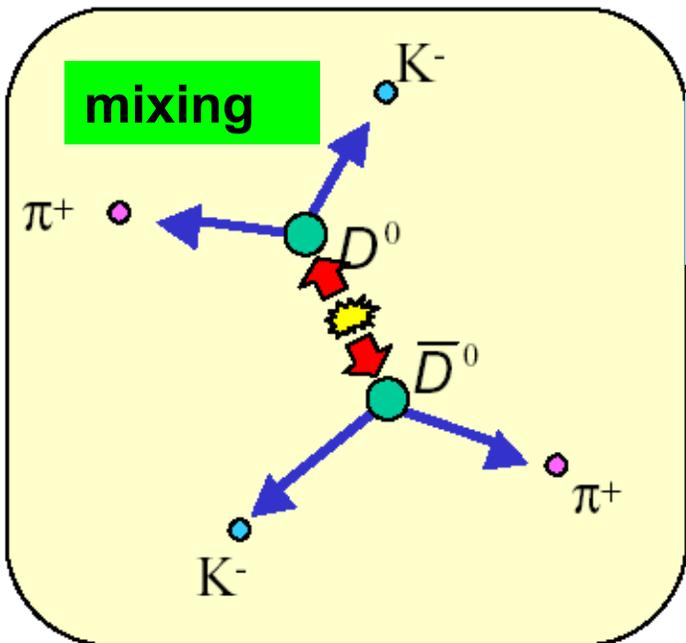
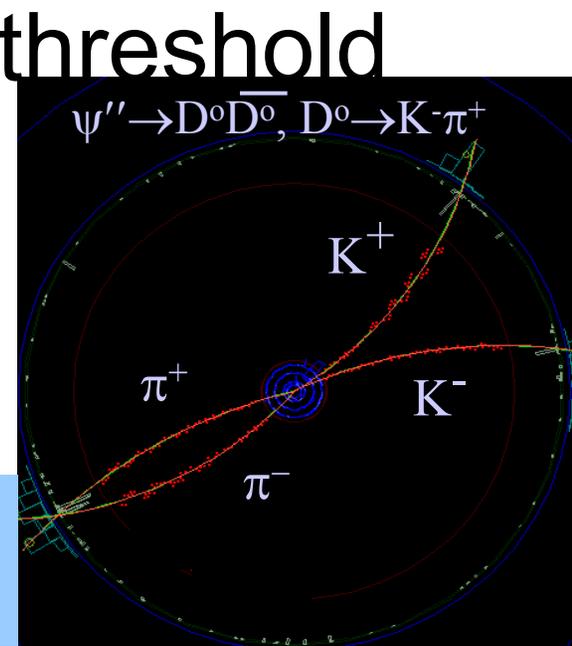
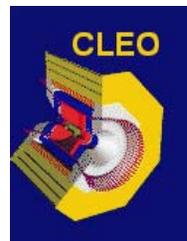
Another D_{sJ}^* (2317) confirmation!



Back to the Future: Charm threshold

Cleo-c and Bes III: Run at $\Psi(3770)$ with high luminosity and a modern detector

Extremely clean events!

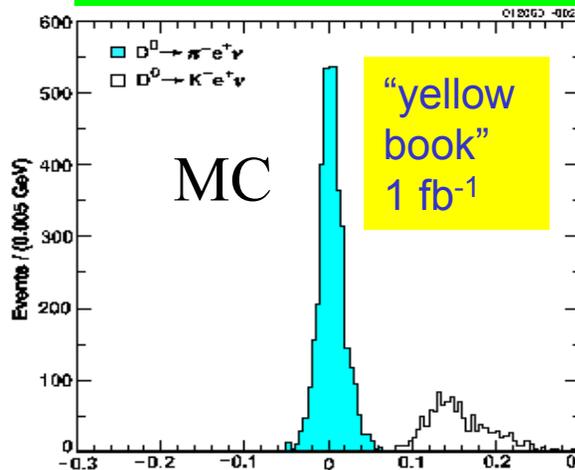


The future of charm is very bright!

Closing the neutrino in $D \rightarrow \pi e \nu$ events using energy momentum balance

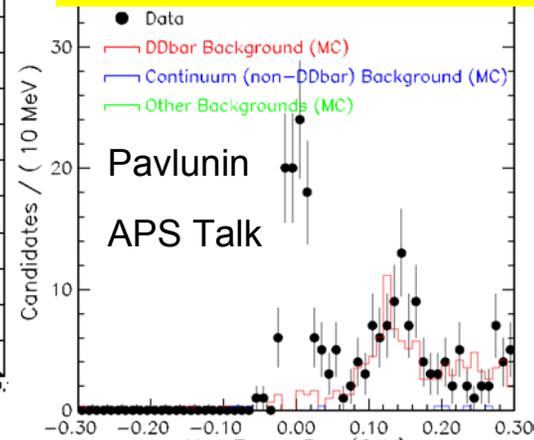
Both D's decay into the same final state

Exploiting quantum coherence



$$U = E_{\text{miss}} - P_{\text{miss}}$$

preliminary data (60 pb⁻¹)



$$U = E_{\text{miss}} - P_{\text{miss}}$$

Mv Notes

Transitions to D0 from L=1

$j_Q \frac{3}{2} \rightarrow \frac{1}{2}$ can couple via

$L_\pi=1$ or $L_\pi=0$ but $L_\pi=1$
violates parity

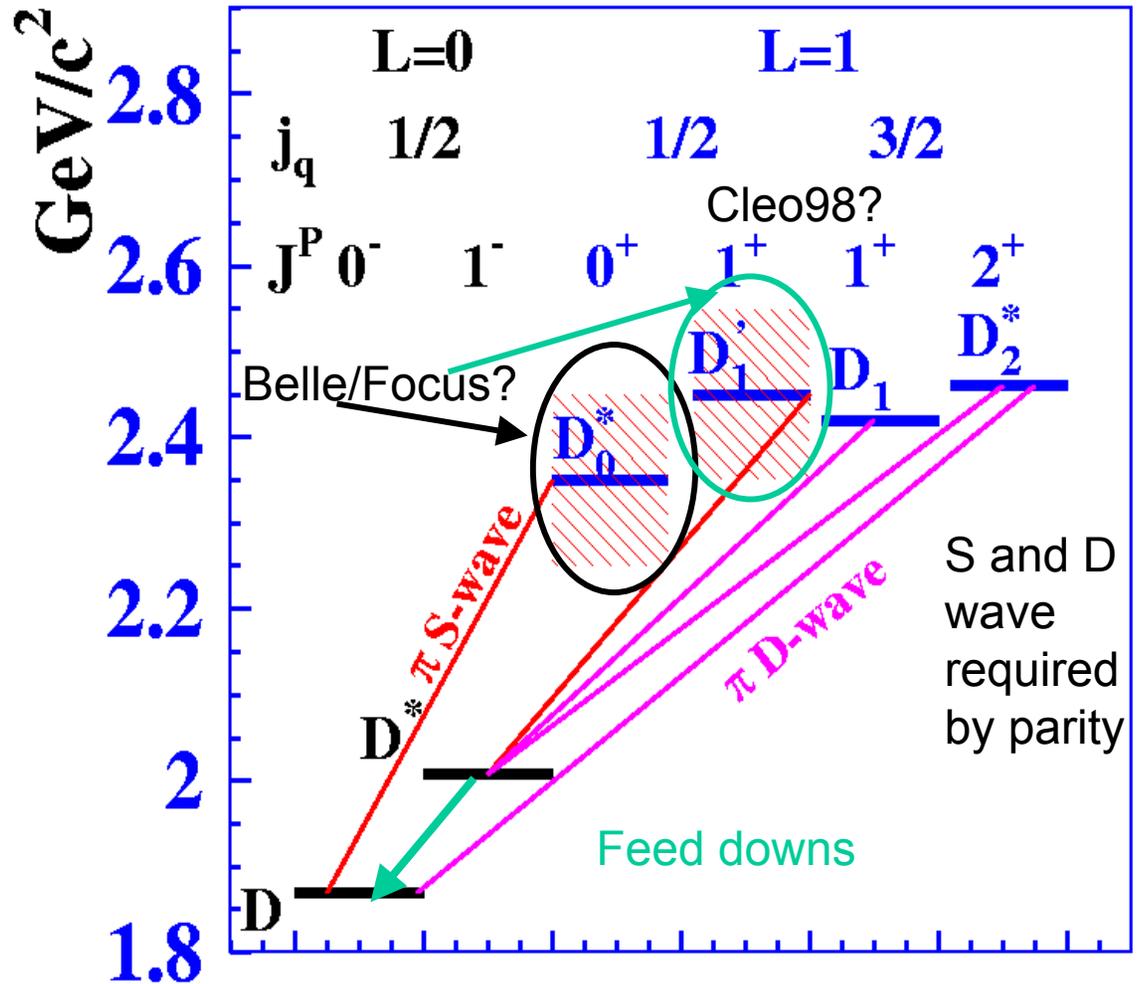
Transitions to D0 from L=1

$j_Q \frac{1}{2} \rightarrow \frac{1}{2}$ can couple via

$L_\pi=0$ or s-wave and go fast

Feed downs are

$D^* \rightarrow \gamma D_0$, $\pi^0 D_0$ or ?

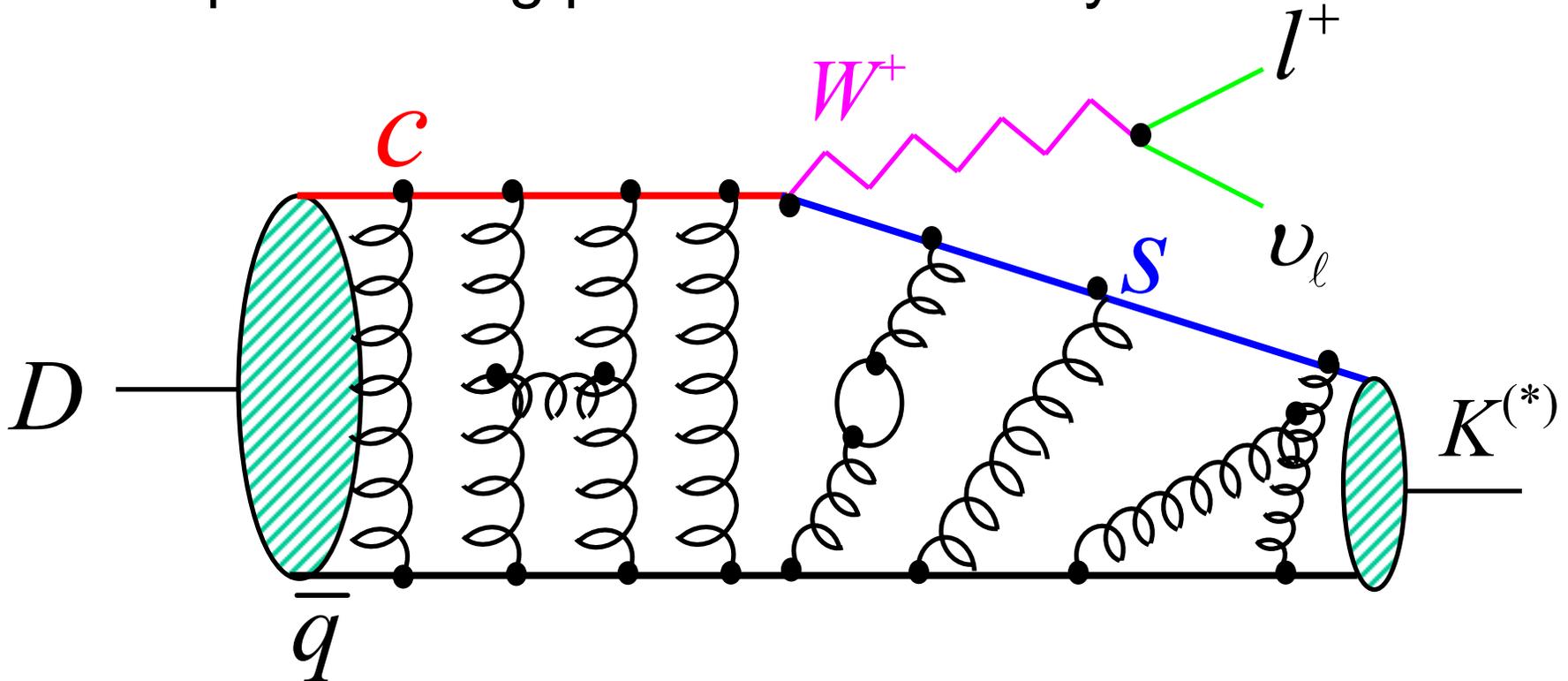


$D_0^* \rightarrow \pi D_0$ but not D_0^* since
L is even

$D_1' \rightarrow \pi D^*$ but not D_0 since
L is even and $L_\pi=0$ is fast

Semileptonic decay as tests of LQCD

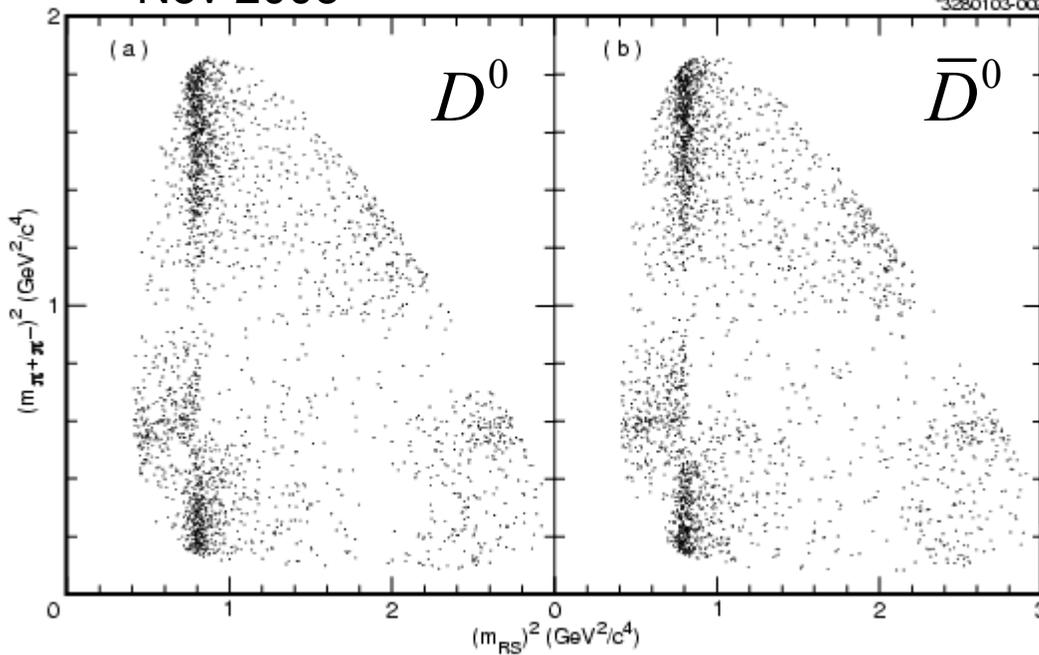
Apart from form factors, these decays can be computed using perturbation theory



The form factors incorporate hadronic complications and can be calculated with non-perturbative Lattice QCD

CP violation in the $K_S \pi^+ \pi^-$ Dalitz plot ?

Nov 2003



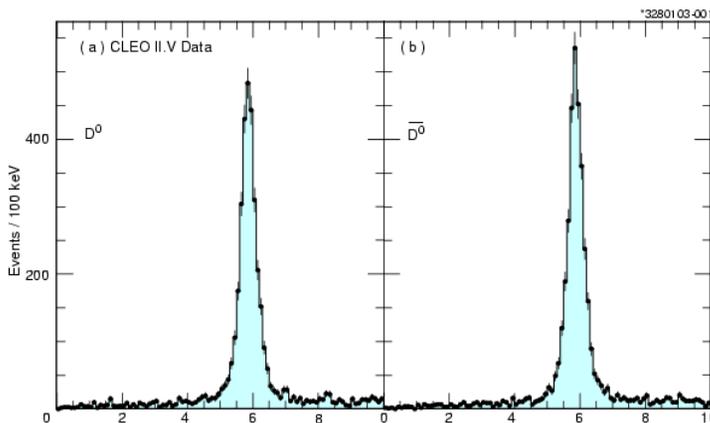
Component	Amplitude (b_j/a_j)
$K^*(892)^+ \pi^- \times B(K^*(892)^+ \rightarrow K^0 \pi^+)$	$-.12 \pm 0.20^{+0.06+0.10}_{-0.15-0.04}$
$\bar{K}^0 \rho^0$	$0.00 \pm 0.02^{+0.02+0.00}_{-0.06-0.04}$
$\bar{K}^0 \omega \times B(\omega \rightarrow \pi^+ \pi^-)$	$-.09 \pm 0.10^{+0.07+0.01}_{-0.00-0.06}$
$K^*(892)^- \pi^+ \times B(K^*(892)^- \rightarrow \bar{K}^0 \pi^-)$	$0.00 \pm 0.02^{+0.01+0.01}_{-0.06-0.05}$
$\bar{K}^0 f_0(980) \times B(f_0(980) \rightarrow \pi^+ \pi^-)$	$-.03 \pm 0.05^{+0.04+0.04}_{-0.06-0.06}$
$\bar{K}^0 f_2(1270) \times B(f_2(1270) \rightarrow \pi^+ \pi^-)$	$0.15 \pm 0.23^{+0.14+0.13}_{-0.19-0.10}$
$\bar{K}^0 f_0(1370) \times B(f_0(1370) \rightarrow \pi^+ \pi^-)$	$0.08 \pm 0.05^{+0.05+0.15}_{-0.15-0.05}$
$K_0^*(1430)^- \pi^+ \times B(K_0^*(1430)^- \rightarrow \bar{K}^0 \pi^-)$	$-.02 \pm 0.05^{+0.07+0.06}_{-0.06-0.06}$
$K_2^*(1430)^- \pi^+ \times B(K_2^*(1430)^- \rightarrow \bar{K}^0 \pi^-)$	$-.06 \pm 0.11^{+0.03+0.11}_{-0.11-0.04}$
$K^*(1680)^- \pi^+ \times B(K^*(1680)^- \rightarrow \bar{K}^0 \pi^-)$	$-.20 \pm 0.09^{+0.11+0.12}_{-0.04-0.24}$

$$A_{CP} = -0.039 \pm 0.034^{+0.014}_{-0.022} \pm 0.027$$

stat exp sys model sys

2579 D0

2720 D0



SM estimates run from 10^{-3} ($\pi^+ \pi^- \pi^0$)
to 10^{-6} for ($K_S^+ \pi^+ \pi^-$)

$$\mathcal{M} = a_0 e^{i\delta_0} + \sum_j a_j e^{i\delta_j} \left(1 + \frac{b_j}{a_j} e^{+i\phi_j}\right) \mathcal{A}_j \quad (1)$$

$$\bar{\mathcal{M}} = a_0 e^{i\delta_0} + \sum_j a_j e^{i\delta_j} \left(1 - \frac{b_j}{a_j} e^{-i\phi_j}\right) \mathcal{A}_j, \quad (2)$$

Probes of D mixing

The time evolution of flavour eigenstates P^0, \bar{P}^0 is given by the Schrödinger equation

$$i \frac{\partial}{\partial t} \begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix}$$

$$H_{jk} \equiv m_{jk} - i\Gamma_{jk} / 2$$

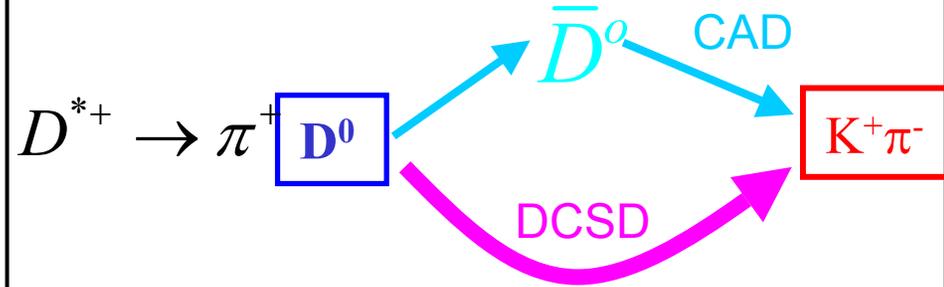
Processes which allow $D^0 \leftrightarrow \bar{D}^0$ mixing appear in H_{12} and H_{21} .

If $H_{12} \neq 0, H_{21} \neq 0$  D^0, \bar{D}^0 are not mass eigenstates. Mass eigenstates are (assuming CP conservation)

$$|D_{1,2}\rangle \equiv \frac{1}{\sqrt{2}} (|D^0\rangle \pm |\bar{D}^0\rangle)$$

Mass and width differences are parametrized by between D_1 and D_2 are given by

$$x \equiv \frac{\Delta m}{\bar{\Gamma}} \quad y \equiv \frac{\Delta \Gamma}{2\bar{\Gamma}}$$



Wrong sign D^* decays $\frac{dN_{D \rightarrow \bar{D}}}{dt} \approx$

$$\left(\left(\frac{x^2 + y^2}{2} \right) \frac{\Gamma^2 t^2}{2} + D(-x \sin \delta + y \cos \delta) \Gamma t + D^2 \right) e^{-\Gamma t}$$

An alternative probe is to directly compare the lifetime of CP eigenstates or mixed eigenstates.

New cu cd L=1 spectroscopy

$\langle D_{1/2} \rangle - D$

Mass	D_2^{*0}	D_2^{*+}		" $D_{1/2}^0$ "	" $D_{1/2}^+$ "
Focus03	2464.5±1.1 ±1.9	2467.6±1.5 ±0.76		2407±21± 35	2403±14 ±35
PDG03	2458.9±2.0	2459±4			
BELLE03	2461.6± 2.1 ± 3.3			2308± 17 ± 32	
Width					
Focus03	38.7±5.3 ±2.9	34.1±6.5 ±4.2		240±55 ±59	283±24 ±34
PDG03	23±5	25 ⁺⁸ ₋₇			
BELLE03	45.6± 4.4 ± 6.6			276±21 ± 66	

Focus	538 ± 39
Kalashnikova 02	564
Di Pierro (2001)	509
Ebert et al. 1998	563
Isgur (1998)	699
Godfrey and Kokoski (1991)	520
Godfrey & Isgur (1985)	520
Eichten et al. (1980)	489
Barbieri + (76)	259
De Rujula + (76)	374

Results roughly agree (1.8σ) between

BELLE [hep-ex/0307021](https://arxiv.org/abs/hep-ex/0307021). FOCUS [hep-ex/0312060](https://arxiv.org/abs/hep-ex/0312060)

Belle measured the spin 1 states as well

	$M D_1$	ΓD_1	$M D_1'$	$\Gamma D_1'$
Belle (03)	2421.4 ± 1.5 ± 0.9	23.7 ± 2.7 ± 4	2427 ± 6 ± 25	384 ± 91 ± 74
Cleo (99)			2461 ± 45	290 ± 110
WA 03	2422 ± 1.8	18.9 ± 4		

